Estimates in subpopulations.

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April 27, 2019

Estimating a mean or total in a subpopulation (domain) from a survey, eg the mean blood pressure in women, is not done simply by taking the subset of data in that subpopulation and pretending it is a new survey. This approach would give correct point estimates but incorrect standard errors.

The standard way to derive domain means is as ratio estimators. I think it is easier to derive them as regression coefficients. These derivations are not important for R users, since subset operations on survey design objects automatically do the necessary adjustments, but they may be of interest. The various ways of constructing domain mean estimators are useful in quality control for the survey package, and some of the examples here are taken from `survey/tests/domain.R`.

Suppose that in the artificial `fpc` data set we want to estimate the mean of \( x \) when \( x > 4 \).

```r
> library(survey)
> data(fpc)
> dfpc<-svydesign(id=~psuid,strat=~stratid,weight=~weight,data=fpc,nest=TRUE)
> dsub<-subset(dfpc,x>4)
> svymean(~x,design=dsub)
```

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>6.195</td>
<td>0.755</td>
</tr>
</tbody>
</table>

The `subset` function constructs a survey design object with information about this subpopulation and `svymean` computes the mean. The same operation can be done for a set of subpopulations with `svyby`.

```r
> svyby(~x,~I(x>4),design=dfpc, svymean)
```

<table>
<thead>
<tr>
<th>I(x &gt; 4)</th>
<th>x</th>
<th>se</th>
</tr>
</thead>
<tbody>
<tr>
<td>FALSE</td>
<td>FALSE</td>
<td>3.314286 0.3117042</td>
</tr>
<tr>
<td>TRUE</td>
<td>TRUE</td>
<td>6.195000 0.7555129</td>
</tr>
</tbody>
</table>

In a regression model with a binary covariate \( Z \) and no intercept, there are two coefficients that estimate the mean of the outcome variable in the subpopulations with \( Z = 0 \) and \( Z = 1 \), so we can construct the domain mean estimator by regression.
> summary(svyglm(x~I(x>4)+0,design=dfpc))

Call:
svyglm(formula = x ~ I(x > 4) + 0, design = dfpc)

Survey design:
svydesign(id = ~psuid, strat = ~stratid, weight = ~weight, data = fpc,
     nest = TRUE)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
I(x > 4)FALSE 3.3143     0.3117 10.630  < 2e-16 ***
I(x > 4)TRUE  6.1950     0.7555  8.200  < 2e-16 ***

---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for gaussian family taken to be 2.557379)

Number of Fisher Scoring iterations: 2

Finally, the classical derivation of the domain mean estimator is as a ratio
where the numerator is $X$ for observations in the domain and 0 otherwise and
the denominator is 1 for observations in the domain and 0 otherwise

> svyratio(~I(x*(x>4)),~as.numeric(x>4), dfpc)

Ratio estimator: svyratio.survey.design2(~I(x * (x > 4)), ~as.numeric(x > 4),
     dfpc)
Ratios=
    as.numeric(x > 4)
I(x * (x > 4))  6.195
SEs=
    as.numeric(x > 4)
I(x * (x > 4))  0.7555129

The estimator is implemented by setting the sampling weight to zero for
observations not in the domain. For most survey design objects this allows a
reduction in memory use, since only the number of zero weights in each sampling
unit needs to be kept. For more complicated survey designs, such as post-
stratified designs, all the data are kept and there is no reduction in memory
use.

More complex examples

Verifying that svymean agrees with the ratio and regression derivations is par-
ticularly useful for more complicated designs where published examples are less
readily available.
This example shows calibration (GREG) estimators of domain means for the California Academic Performance Index (API).

```r
> data(api)
> dclus1 <- svydesign(id=~dnum, weights=~pw, data=apiclus1, fpc=~fpc)
> pop.totals<-c(~(Intercept)~ = 6194, stypeH = 755, stypeM = 1018)
> gclus1 <- calibrate(dclus1, ~stype+api99, c(pop.totals, api99 = 3914069))
> svymean(~api00, subset(gclus1, comp.imp == "Yes"))

      mean      SE
api00 672.05 6.5182

> svyratio(~I(api00*(comp.imp="Yes")), ~as.numeric(comp.imp="Yes"), gclus1)

Ratios = 

[1] as.numeric(comp.imp == "Yes")

672.049

SEs = 

[1] as.numeric(comp.imp == "Yes")

6.518153

> summary(svyglm(api00~comp.imp-1, gclus1))

Call: 
svyglm(formula = api00 ~ comp.imp - 1, design = gclus1)

Survey design:
calibrate(dclus1, ~stype + api99, c(pop.totals, api99 = 3914069))

Coefficients:

                     Estimate Std. Error t value Pr(>|t|)
comp.impNo         649.706   12.563   51.72 <2e-16 ***
comp.impYes        672.049   6.518  103.10 <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for gaussian family taken to be 10519.86)

Number of Fisher Scoring iterations: 2

Two-stage samples with full finite-population corrections
```
Ratio estimator: svyratio.survey.design2(~I(y1 * (y1 > 40)), ~as.numeric(y1 > 40), dmu284)
Ratios=

<table>
<thead>
<tr>
<th>I(y1 * (y1 &gt; 40))</th>
<th>48.69014</th>
</tr>
</thead>
</table>

SEs=

<table>
<thead>
<tr>
<th>I(y1 * (y1 &gt; 40))</th>
<th>1.108825</th>
</tr>
</thead>
</table>

> summary(svyglm(y1~I(y1>40)+0, dmu284))

Call:
svyglm(formula = y1 ~ I(y1 > 40) + 0, design = dmu284)

Survey design:
svydesign(id = ~id1 + id2, fpc = ~n1 + n2, data = mu284)

Coefficients:

| Estimate | Std. Error | t value | Pr(> |t|) |
|----------|------------|---------|------|-----|
| I(y1 > 40)FALSE | 34.419 | 1.842 | 18.69 | 0.000334 *** |
| I(y1 > 40)TRUE | 48.690 | 1.109 | 43.91 | 2.6e-05 *** |

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Signif. codes: 0 âŠ™â™¥â™¥â™¾ 0.001 âŠ™â™¥â™¥â™° 0.01 âŠ™â™°â™¾ 0.05 âŠ™â™¾ â™½ 1

(Dispersion parameter for gaussian family taken to be 27.96987)

Number of Fisher Scoring iterations: 2

Stratified two-phase sampling of children with Wilm’s Tumor, estimating relapse probability for those older than 3 years (36 months) at diagnosis

> library("survival")
> data(nwtco)
> nwtco$incc2<-as.logical(with(nwtco, ifelse(rel | instit==2,1,rbinom(nrow(nwtco),1,.1))))
> dccs8<-twophase(id=list(~seqno,~seqno), strata=list(NULL,~interaction(rel,stage,instit)), +   data=nwtco, subset=~incc2)
> svymean(~rel, subset(dccs8, age>36))

<table>
<thead>
<tr>
<th>mean</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>rel</td>
<td>0.16526</td>
</tr>
</tbody>
</table>

> svyratio(~I(rel*as.numeric(age>36)), ~as.numeric(age>36), dccs8)
Ratio estimator: svyratio.twophase2(\(I(\text{rel} * \text{as.numeric(age > 36)})\), \(\text{as.numeric(age > 36)}\), dccs8)

Ratios=

\(\text{as.numeric(age > 36)}\) 0.1652602

SEs=

\(\text{as.numeric(age > 36)}\) 0.01020621

\(> \text{summary(svyglm(rel~I(age>36)+0, dccs8))}\)

Call:

svyglm(formula = rel ~ I(age > 36) + 0, design = dccs8)

Survey design:
twophase2(id = id, strata = strata, probs = probs, fpc = fpc, subset = subset, data = data)

Coefficients:                        Estimate  Std. Error t value  Pr(>|t|)
I(age > 36)FALSE 0.114901  0.008906 12.90  <2e-16 ***
I(age > 36)TRUE  0.165260  0.010206 16.19  <2e-16 ***
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 0.1211382)

Number of Fisher Scoring iterations: 2