

# Strogatz's model

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## Model

The function `strogatz` simulates Strogatz's model for sleep-wake dynamics. The dynamics have the following structure:

$$\dot{\theta}_1 = \omega_1 - C_1 \cdot \cos(2\pi(\theta_2 - \theta_1)) \quad \dot{\theta}_2 = \omega_2 + C_2 \cdot \cos(2\pi(\theta_2 - \theta_1))$$

## Analysis

It is useful to note that, using the relative phase  $\psi \equiv \theta_1 - \theta_2$ , the system can be simplified to:

$$\dot{\psi} = \Omega - C \cos(2\pi\psi)$$

where  $\Omega \equiv \omega_1 - \omega_2$  and  $C \equiv C_1 + C_2$ . The relative phase can be stabilized if and only if  $\dot{\psi} = 0$ , and this is only possible if:

$$\left| \frac{\Omega}{C} \right| \leq 1$$

## Reference

Strogatz, S. H. (1987). Human sleep and circadian rhythms: a simple model based on two coupled oscillators. *Journal of Mathematical Biology*, 25(3), 327–347. <http://doi.org/10.1007/BF00276440>

## Examples of usage

### Getting the time series

With default parameters:

```
## Problem setting
y0 <- c(th1 = 0.1, th2 = 0.05) # Initial conditions

nDays <- 5
ts <- seq(0, nDays*24, length.out=nDays*24*20) # Times to simulate

# Simulate
sol <- strogatz(ts, y0)
```

With custom parameters:

```
## Problem setting
y0 <- c(th1 = 0.1, th2 = 0.05) # Initial conditions

nDays <- 5
ts <- seq(0, nDays*24, length.out=nDays*24*20) # Times to simulate
parms <- c(w1=1/24, w2=0.85/24, C1=0/24, C2=0.16/24) # Parameters

# Simulate
sol <- strogatz(ts, y0, parms)
```

The output looks like:

time	th1	th2	asleep
0.0000000	0.1000000	0.0500000	TRUE
0.0500208	0.1020842	0.0520887	TRUE
0.1000417	0.1041684	0.0541775	TRUE
0.1500625	0.1062526	0.0562662	TRUE
0.2000834	0.1083368	0.0583549	TRUE

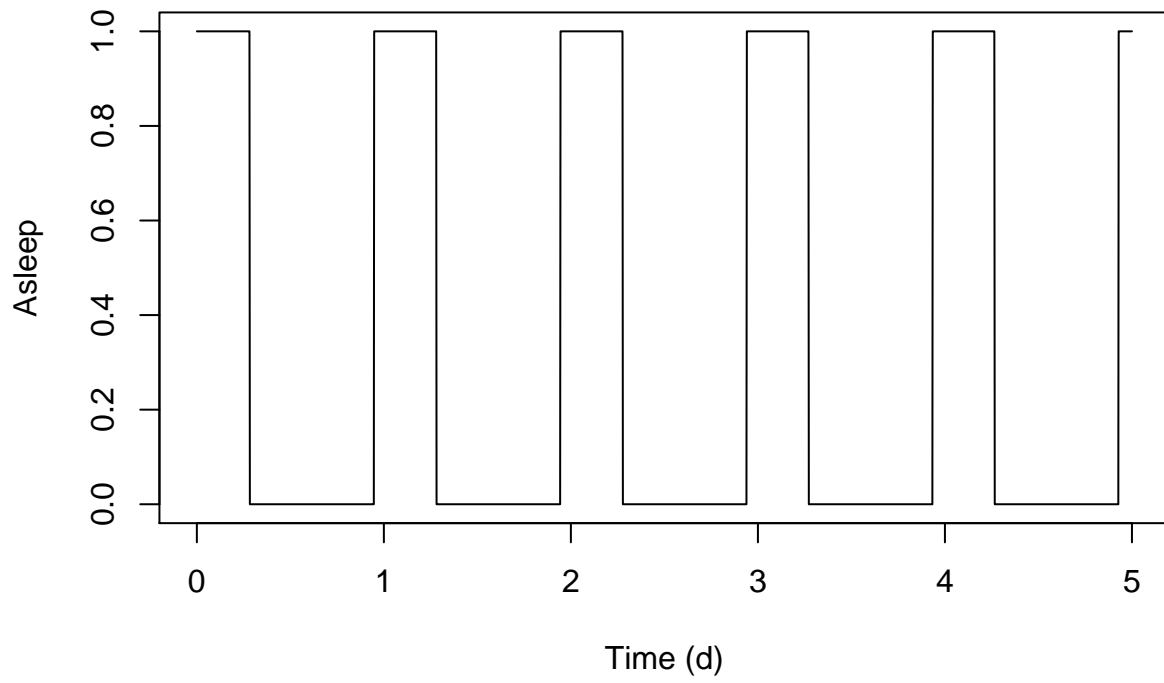
where:

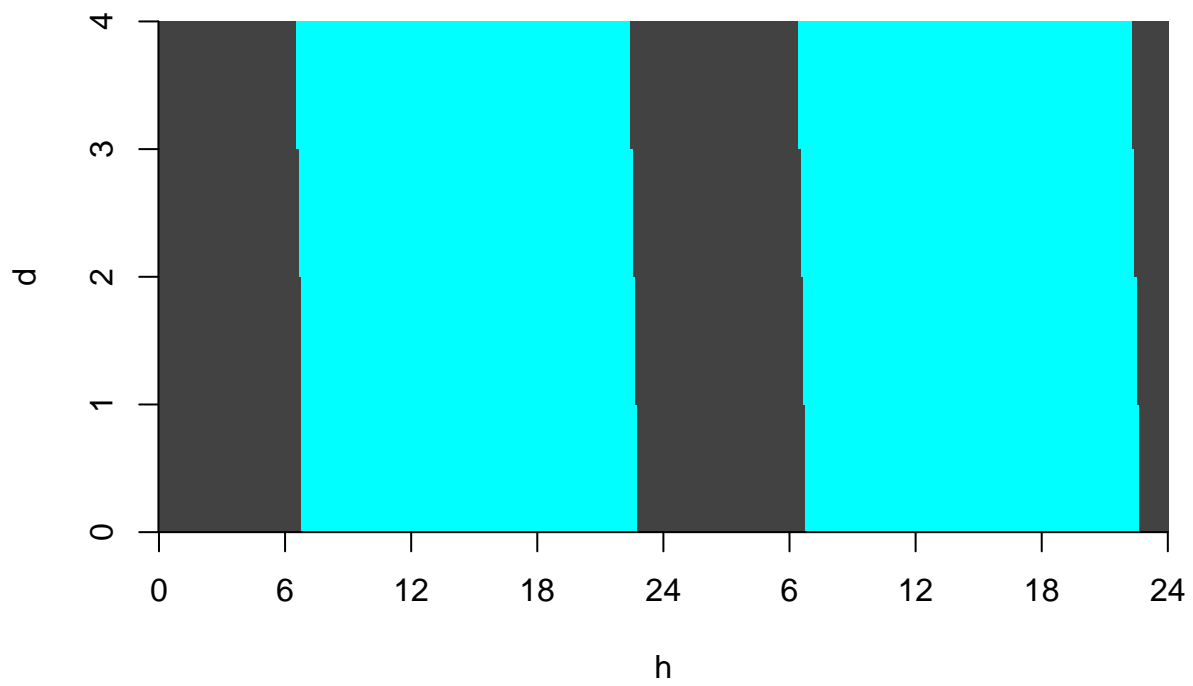
- **time**: the time (in h),
- **th1**: the phase of the circadian oscillator
- **th2**: the phase of the sleep-wake cycle
- **asleep**: the asleep/awake status (TRUE if asleep, FALSE if awake)

### Plotting results

Raster / somnogram plot

```
plot(sol$time/24, sol$asleep, type = 'line', xlab = 'Time (d)', ylab = 'Asleep')
rasterPlot(sol)
```

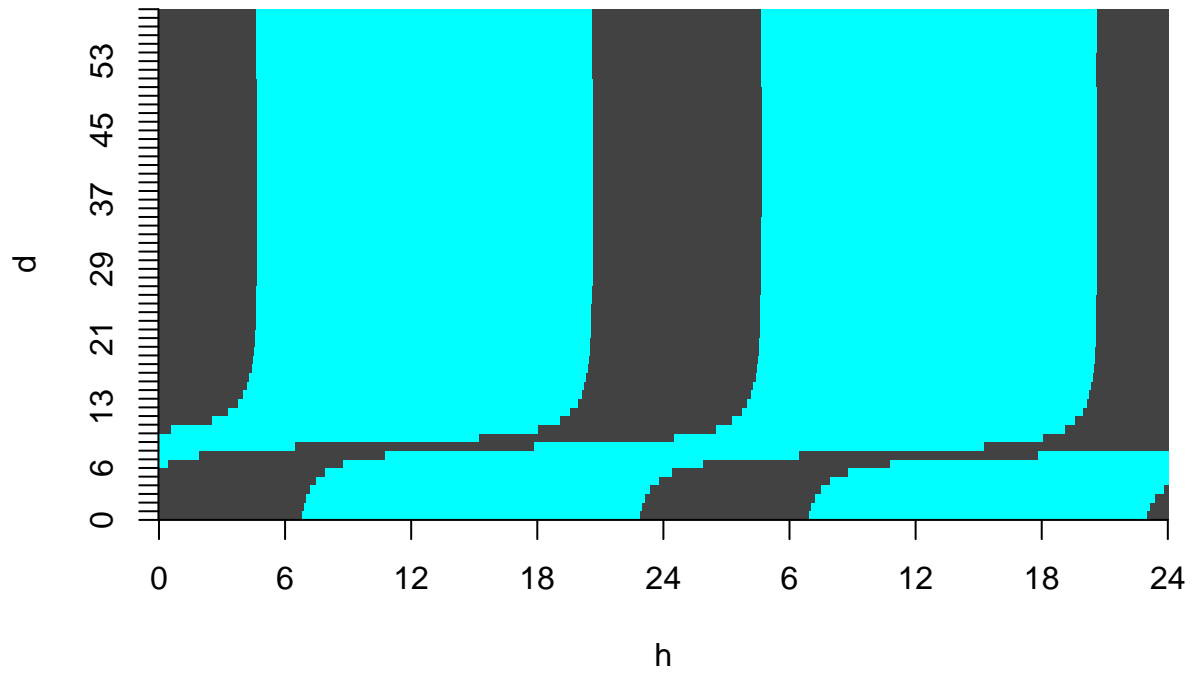




# Results

Entrained vs. non entrained case

### Entrained



### Not entrained

