The **libcoin** Package

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Contents

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Chapter 1

Introduction

The **libcoin** package implements a generic framework for permutation tests. We assume that we are provided with n observations

$$(\mathbf{Y}_i, \mathbf{X}_i, w_i, \text{block}_i), \quad i = 1, \dots, N.$$

The variables $\mathbf{Y}$ and $\mathbf{X}$ from sample spaces $\mathcal{Y}$ and $\mathcal{X}$ may be measured at arbitrary scales and may be multivariate as well. In addition to those measurements, case weights $w_i \in \mathbb{N}$ and a factor $\text{block}_i \in \{1, \dots, B\}$ coding for B independent blocks may be available. We are interested in testing the null hypothesis of independence of $\mathbf{Y}$ and $\mathbf{X}$

$$H_0 : D(\mathbf{Y} \mid \mathbf{X}) = D(\mathbf{Y})$$

against arbitrary alternatives. Strasser and Weber (1999) suggest to derive scalar test statistics for testing H_0 from multivariate linear statistics of a specific linear form. Let $\mathcal{A} \subseteq \{1, \dots, N\}$ denote some subset of the observation numbers and consider the linear statistic

$$\mathbf{T}(\mathcal{A}) = \text{vec}\left(\sum_{i \in \mathcal{A}} w_i g(\mathbf{X}_i) h(\mathbf{Y}_i, \{\mathbf{Y}_i \mid i \in \mathcal{A}\})^\top\right) \in \mathbb{R}^{pq}. \tag{1.1}$$

Here, $g : \mathcal{X} \to \mathbb{R}^P$ is a transformation of $\mathbf{X}$ known as the *regression function* and $h : \mathcal{Y} \times \mathcal{Y}^n \to \mathbb{R}^Q$ is a transformation of $\mathbf{Y}$ known as the *influence function*, where the latter may depend on $\mathbf{Y}_i$ for $i \in \mathcal{A}$ in a permutation symmetric way. We will give specific examples on how to choose g and h later on.

With $\mathbf{x}_i = g(\mathbf{X}_i) \in \mathbb{R}^P$ and $\mathbf{y}_i = h(\mathbf{Y}_i, \{\mathbf{Y}_i, i \in \mathcal{A}\}) \in \mathbb{R}^Q$ we write

$$\mathbf{T}(\mathcal{A}) = \text{vec}\left(\sum_{i \in \mathcal{A}} w_i \mathbf{x}_i \mathbf{y}_i^\top\right) \in \mathbb{R}^{PQ}. \tag{1.2}$$

The **libcoin** package doesn't handle neither g nor h, this is the job of **coin** and we therefore continue with $\mathbf{x}_i$ and $\mathbf{y}_i$.

The distribution of $\mathbf{T}$ depends on the joint distribution of $\mathbf{Y}$ and $\mathbf{X}$, which is unknown under almost all practical circumstances. At least under the null hypothesis one can dispose of this dependency by fixing $\mathbf{X}_i, i \in \mathcal{A}$ and conditioning on all possible permutations $S(\mathcal{A})$ of the responses $\mathbf{Y}_i, i \in \mathcal{A}$. This principle leads to test procedures known as *permutation tests*. The conditional expectation $\mu(\mathcal{A}) \in \mathbb{R}^{PQ}$ and covariance $\Sigma(\mathcal{A}) \in \mathbb{R}^{PQ \times PQ}$ of $\mathbf{T}$ under H_0 given all permutations $\sigma \in S(\mathcal{A})$ of the responses are derived by Strasser and Weber (1999):

$$
\begin{aligned}
\mu(\mathcal{A}) &= \mathbb{E}(\mathbf{T}(\mathcal{A}) \mid S(\mathcal{A})) = \operatorname{vec}\left(\left(\sum_{i \in \mathcal{A}} w_i \mathbf{x}_i\right) \mathbb{E}(h \mid S(\mathcal{A}))^\top\right), \\
\Sigma(\mathcal{A}) &= \mathbb{V}(\mathbf{T}(\mathcal{A}) \mid S(\mathcal{A})) \\
&= \frac{\mathbf{w}.}{\mathbf{w}.(\mathcal{A}) - 1} \mathbb{V}(h \mid S(\mathcal{A})) \otimes \left(\sum_{i \in \mathcal{A}} w_i \mathbf{x}_i \otimes w_i \mathbf{x}_i^\top\right) \\
&- \frac{1}{\mathbf{w}.(\mathcal{A}) - 1} \mathbb{V}(h \mid S(\mathcal{A})) \otimes \left(\sum_{i \in \mathcal{A}} w_i \mathbf{x}_i\right) \otimes \left(\sum_{i \in \mathcal{A}} w_i \mathbf{x}_i\right)^\top
\end{aligned} \tag{1.3}
$$

where $\mathbf{w}.(\mathcal{A}) = \sum_{i \in \mathcal{A}} w_i$ denotes the sum of the case weights, and $\otimes$ is the Kronecker product. The conditional expectation of the influence function is

$$
\mathbb{E}(h \mid S(\mathcal{A})) = \mathbf{w}.(\mathcal{A})^{-1} \sum_{i \in \mathcal{A}} w_i \mathbf{y}_i \in \mathbb{R}^Q
$$

with corresponding $Q \times Q$ covariance matrix

$$
\mathbb{V}(h \mid S(\mathcal{A})) = \mathbf{w}.(\mathcal{A})^{-1} \sum_{i \in \mathcal{A}} w_i \left(\mathbf{y}_i - \mathbb{E}(h \mid S(\mathcal{A}))\right) \left(\mathbf{y}_i - \mathbb{E}(h \mid S(\mathcal{A}))\right)^\top.
$$

With $A_b = \{i \mid \text{block}_i = b\}$ we get $\mathbf{T} = \sum_{b=1}^{B} T(\mathcal{A}_b)$, $\mu = \sum_{b=1}^{B} \mu(\mathcal{A}_b)$ and $\Sigma = \sum_{b=1}^{B} \Sigma(\mathcal{A}_b)$.

Having the conditional expectation and covariance at hand we are able to standardize a linear statistic $\mathbf{T} \in \mathbb{R}^{PQ}$ of the form (1.2). Univariate test statistics c mapping an observed linear statistic $\mathbf{t} \in \mathbb{R}^{PQ}$ into the real line can be of arbitrary form. An obvious choice is the maximum of the absolute values of the standardized linear statistic

$$
c_{\max}(\mathbf{t}, \mu, \Sigma) = \max \left| \frac{\mathbf{t} - \mu}{\operatorname{diag}(\Sigma)^{1/2}} \right|
$$

utilizing the conditional expectation μ and covariance matrix Σ. The application of a quadratic form $c_{\text{quad}}(\mathbf{t}, \mu, \Sigma) = (\mathbf{t} - \mu)\Sigma^+(\mathbf{t} - \mu)^\top$ is one alternative, although computationally more expensive because the Moore-Penrose inverse Σ^+ of Σ is involved.

The definition of one- and two-sided p-values used for the computations in the **libcoin** package is

$$
\begin{aligned}
P(c(\mathbf{T}, \mu, \Sigma) &\le c(\mathbf{t}, \mu, \Sigma)) \quad \text{(less)} \\
P(c(\mathbf{T}, \mu, \Sigma) &\ge c(\mathbf{t}, \mu, \Sigma)) \quad \text{(greater)} \\
P(|c(\mathbf{T}, \mu, \Sigma)| &\le |c(\mathbf{t}, \mu, \Sigma)|) \quad \text{(two-sided)}.
\end{aligned}
$$

Note that for quadratic forms only two-sided p-values are available and that in the one-sided case maximum type test statistics are replaced by

$$
\min\left(\frac{\mathbf{t} - \mu}{\operatorname{diag}(\Sigma)^{1/2}}\right) \quad \text{(less) and } \max\left(\frac{\mathbf{t} - \mu}{\operatorname{diag}(\Sigma)^{1/2}}\right) \quad \text{(greater)}.
$$

This single source file implements and documents the **libcoin** package following the literate programming paradigm. The keynote lecture on literate programming by Donald E. Knuth given at useR! 2016 in Stanford very much motivated this little experiment.

Chapter 2

R Code

2.1 R User Interface

"libcoin.R" 3a≡

⟨ *R Header* 166a ⟩
⟨ *LinStatExpCov* 4 ⟩
⟨ *LinStatExpCov1d* 6 ⟩
⟨ *LinStatExpCov2d* 8 ⟩
⟨ *vcov LinStatExpCov* 10 ⟩
⟨ *doTest* 12 ⟩
⟨ *Contrasts* 14 ⟩
◇

The **libcoin** package implements two functions, `LinStatExpCov` and `doTest` for the computation of linear statistics, their expectation and covariance as well as for the computation of test statistics and p-values. There are two interfaces: One (labelled "1d") when the data is available as matrices `X` and `Y`, both with the same number of rows N. The second interface (labelled "2d") handles the case when the data is available in aggregated form; details will be explained later.

⟨ *LinStatExpCov Prototype* 3b ⟩ ≡

```
(X, Y, ix = NULL, iy = NULL, weights = integer(0),
 subset = integer(0), block = integer(0), checkNAs = TRUE,
 varonly = FALSE, nresample = 0, standardise = FALSE,
 tol = sqrt(.Machine$double.eps))◇
```

Fragment referenced in 4, 18.
Uses: `block` 28bd, `subset` 27be, 28a, `weights` 26c.

⟨ *LinStatExpCov* 4 ⟩ ≡

```
LinStatExpCov <-
function⟨ LinStatExpCov Prototype 3b ⟩
{
    if (missing(X) & !is.null(ix) & is.null(iy)) {
        X <- ix
        ix <- NULL
    }

    if (missing(X)) X <- integer(0)

    ## <FIXME> for the time being only!!! </FIXME>
##    if (length(subset) > 0) subset <- sort(subset)

    if (is.null(ix) & is.null(iy))
        return(.LinStatExpCov1d(X = X, Y = Y, weights = weights,
                                subset = subset, block = block,
                                checkNAs = checkNAs,
                                varonly = varonly, nresample = nresample,
                                standardise = standardise, tol = tol))

    if (!is.null(ix) & !is.null(iy))
        return(.LinStatExpCov2d(X = X, Y = Y, ix = ix, iy = iy,
                                weights = weights, subset = subset,
                                block = block, varonly = varonly,
                                checkNAs = checkNAs, nresample = nresample,
                                standardise = standardise, tol = tol))

    stop("incorrect call to LinStatExpCov")
}
◇
```

Fragment referenced in 3a.
Uses: `block` 28bd, `subset` 27be, 28a, `weights` 26c, `weights,` 26de.

2.1.1 One-Dimensional Case ("1d")

We assume that $\mathbf{x}_i$ and $\mathbf{y}_i$ for $i = 1, \dots, N$ are available as numeric matrices `X` and `Y` with N rows as well as P and Q columns, respectively. The special case of a dummy matrix `X` with P columns can also be represented by a factor at P levels. The vector of case weights `weights` can be stored as `integer` or `double` (possibly resulting from an aggregation of $N >$ `INT_MAX` observations). The subset vector `subset` may contain the elements $1, \dots, N$ as `integer` or `double` (for $N >$ `INT_MAX`) and can be longer than N. The `subset` vector MUST be sorted. `block` is a factor at B levels of length N.

⟨ *Check weights, subset, block* 5a ⟩ ≡

```
if (is.null(weights)) weights <- integer(0)

if (length(weights) > 0) {
    if (!((N == length(weights)) && all(weights >= 0)))
        stop("incorrect weights")
    if (checkNAs) stopifnot(!anyNA(weights))
}

if (is.null(subset)) subset <- integer(0)

if (length(subset) > 0 && checkNAs) {
    rs <- range(subset)
    if (anyNA(rs)) stop("no missing values allowed in subset")
    if (!((rs[2] <= N) && (rs[1] >= 1L)))
        stop("incorrect subset")
}

if (is.null(block)) block <- integer(0)

if (length(block) > 0) {
    if (!((N == length(block)) && is.factor(block)))
        stop("incorrect block")
    if (checkNAs) stopifnot(!anyNA(block))
}
◇
```

Fragment referenced in 6, 8, 16.
Uses: `block` 28bd, `N` 24bc, `subset` 27be, 28a, `weights` 26c.

Missing values are only allowed in `X` and `Y`, all other vectors must not contain `NA`s. Missing values are dealt with by excluding the corresponding observations from the subset vector.

⟨ *Handle Missing Values* 5b ⟩ ≡

```
ms <- !complete.cases(X, Y)
if (all(ms))
    stop("all observations are missing")
if (any(ms)) {
    if (length(subset) > 0) {
        if (all(subset %in% which(ms)))
            stop("all observations are missing")
        subset <- subset[!(subset %in% which(ms))]
    } else {
        subset <- (1:N)[-which(ms)]
    }
}
◇
```

Fragment referenced in 6.
Uses: `N` 24bc, `subset` 27be, 28a.

The logical argument `varonly` triggers the computation of the diagonal elements of the covariance matrix Σ only. `nresample` permuted linear statistics under the null hypothesis H_0 are returned on the original and standardised scale (the latter only when `standardise` is `TRUE`). Variances smaller than `tol` are treated as being zero.

⟨ *LinStatExpCov1d* 6 ⟩ ≡

```
.LinStatExpCov1d <-
function(X, Y, weights = integer(0), subset = integer(0), block = integer(0),
         checkNAs = TRUE, varonly = FALSE, nresample = 0, standardise = FALSE,
         tol = sqrt(.Machine$double.eps))
{
    if (NROW(X) != NROW(Y))
        stop("dimensions of X and Y don't match")
    N <- NROW(X)

    if (is.integer(X)) {
        if (is.null(attr(X, "levels")) || checkNAs) {
            rg <- range(X)
            if (anyNA(rg))
                stop("no missing values allowed in X")
            stopifnot(rg[1] > 0) # no missing values allowed here!
            if (is.null(attr(X, "levels")))
                attr(X, "levels") <- 1:rg[2]
        }
    }

    if (is.factor(X) && checkNAs)
        stopifnot(!anyNA(X))

    ⟨ Check weights, subset, block 5a ⟩

    if (checkNAs) {
        ⟨ Handle Missing Values 5b ⟩
    }

    ret <- .Call(R_ExpectationCovarianceStatistic, X, Y, weights, subset,
                 block, as.integer(varonly), as.double(tol))
    ret$varonly <- as.logical(ret$varonly)
    ret$Xfactor <- as.logical(ret$Xfactor)
    if (nresample > 0) {
        ret$PermutedLinearStatistic <-
            .Call(R_PermutedLinearStatistic, X, Y, weights, subset,
                  block, as.double(nresample))
        if (standardise)
            ret$StandardisedPermutedLinearStatistic <-
                .Call(R_StandardisePermutedLinearStatistic, ret)
    }
    class(ret) <- c("LinStatExpCov1d", "LinStatExpCov")
    ret
}
◇
```

Fragment referenced in 3a.
Uses: `block` 28bd, `N` 24bc, `NROW` 139b, `R_ExpectationCovarianceStatistic` 32c, `R_PermutedLinearStatistic` 40, `subset` 27be, 28a, `weights` 26c, `weights,` 26de.

Here is a simple example. We have five groups and a uniform outcome (rounded to one digit) and want to test independence of group membership and outcome. The simplest way is to set-up the dummy matrix explicitly:

```
> isequal <- function(a, b) {
+     attributes(a) <- NULL
```

```
+     attributes(b) <- NULL
+     if (!isTRUE(all.equal(a, b))) {
+         print(a, digits = 10)
+         print(b, digits = 10)
+         FALSE
+     } else
+         TRUE
+ }
> library("libcoin")
> set.seed(290875)
> x <- gl(5, 20)
> y <- round(runif(length(x)), 1)
> ls1 <- LinStatExpCov(X = model.matrix(~ x - 1), Y = matrix(y, ncol = 1))
> ls1$LinearStatistic
```

```
[1]  8.8  9.5 10.3  9.8 10.5
```

```
> tapply(y, x, sum)
```

```
   1    2    3    4    5 
 8.8  9.5 10.3  9.8 10.5 
```

The linear statistic is simply the sum of the response in each group. Alternatively, we can compute the same object without setting-up the dummy matrix:

```
> ls2 <- LinStatExpCov(X = x, Y = matrix(y, ncol = 1))
> all.equal(ls1[-grep("Xfactor", names(ls1))],
+           ls2[-grep("Xfactor", names(ls2))])
```

```
[1] TRUE
```

The results are identical, except for a logical indicating that a factor was used to represent the dummy matrix `X`.

2.1.2 Two-Dimensional Case ("2d")

Sometimes the data takes only a few unique values and considerable computational speedups can be achieved taking this information into account. Let `ix` denote an integer vector with elements $0, \dots, L_x$ of length N and `iy` an integer vector with elements $0, \dots, L_y$, also of length N. The matrix `X` is now of dimension $(L_x + 1) \times P$ and the matrix `Y` of dimension $(L_y + 1) \times Q$. The combination of `X` and `ix` means that the ith observation corresponds to the row `X[ix[i] + 1,]`. This looks cumbersome in R notation but is a very efficient way of dealing with missing values at C level. By convention, elements of `ix` being zero denote a missing value (`NA`s are not allowed in `ix` and `iy`). Thus, the first row of `X` corresponds to a missing value. If the first row is simply zero, missing values do not contribute to any of the sums computed later. Even more important is the fact that all entities, such as linear statistics etc., can be computed from the two-way tabulation (therefore the abbrevation "2d") over the N elements of `ix` and `iy`. Once such a table was computed, the remaining computations can be performed in dimension $L_x \times L_y$, typically much smaller than N.

⟨ *LinStatExpCov2d* 8 ⟩ ≡

```
.LinStatExpCov2d <-
function(X = numeric(0), Y, ix, iy, weights = integer(0), subset = integer(0),
         block = integer(0), checkNAs = TRUE, varonly = FALSE, nresample = 0,
         standardise = FALSE, tol = sqrt(.Machine$double.eps))
{
    IF <- function(x) is.integer(x) || is.factor(x)

    if (!((length(ix) == length(iy)) && IF(ix) && IF(iy)))
        stop("incorrect ix and/or iy")
    N <- length(ix)

    ⟨ Check ix 9a ⟩

    ⟨ Check iy 9b ⟩

    if (length(X) > 0) {
        if (!(NROW(X) == (length(attr(ix, "levels")) + 1) &&
              all(complete.cases(X))))
            stop("incorrect X")
    }

    if (!(NROW(Y) == (length(attr(iy, "levels")) + 1) &&
          all(complete.cases(Y))))
        stop("incorrect Y")

    ⟨ Check weights, subset, block 5a ⟩

    ret <- .Call(R_ExpectationCovarianceStatistic_2d, X, ix, Y, iy,
                 weights, subset, block, as.integer(varonly), as.double(tol))
    ret$varonly <- as.logical(ret$varonly)
    ret$Xfactor <- as.logical(ret$Xfactor)
    if (nresample > 0) {
        ret$PermutedLinearStatistic <-
            .Call(R_PermutedLinearStatistic_2d, X, ix, Y, iy, block, nresample, ret$Table)
        if (standardise)
            ret$StandardisedPermutedLinearStatistic <-
                .Call(R_StandardisePermutedLinearStatistic, ret)
    }
    class(ret) <- c("LinStatExpCov2d", "LinStatExpCov")
    ret
}
◇
```

Fragment referenced in 3a.
Uses: `block` 28bd, `N` 24bc, `NROW` 139b, `R_ExpectationCovarianceStatistic_2d` 44, `R_PermutedLinearStatistic_2d` 51, `subset` 27be, 28a, `weights` 26c, `weights`, 26de, `x` 24d, 25bc.

`ix` can be a factor but without any missing values

⟨ *Check ix* 9a ⟩ ≡

```
if (is.null(attr(ix, "levels"))) {
    rg <- range(ix)
    if (anyNA(rg))
        stop("no missing values allowed in ix")
    stopifnot(rg[1] >= 0)
    attr(ix, "levels") <- 1:rg[2]
} else {
    ## lev can be data.frame (see inum::inum)
    lev <- attr(ix, "levels")
    if (!is.vector(lev)) lev <- 1:NROW(lev)
    attr(ix, "levels") <- lev
    if (checkNAs) stopifnot(!anyNA(ix))
}
◇
```

Fragment referenced in 8, 16.
Uses: `NROW` 139b.

⟨ *Check iy* 9b ⟩ ≡

```
if (is.null(attr(iy, "levels"))) {
    rg <- range(iy)
    if (anyNA(rg))
        stop("no missing values allowed in iy")
    stopifnot(rg[1] >= 0)
    attr(iy, "levels") <- 1:rg[2]
} else {
    ## lev can be data.frame (see inum::inum)
    lev <- attr(iy, "levels")
    if (!is.vector(lev)) lev <- 1:NROW(lev)
    attr(iy, "levels") <- lev
    if (checkNAs) stopifnot(!anyNA(iy))
}
◇
```

Fragment referenced in 8, 16.
Uses: `NROW` 139b.

In our small example, we can set-up the data in the following way

```
> X <- rbind(0, diag(nlevels(x)))
> ix <- unclass(x)
> ylev <- sort(unique(y))
> Y <- rbind(0, matrix(ylev, ncol = 1))
> iy <- .bincode(y, breaks = c(-Inf, ylev, Inf))
> ls3 <- LinStatExpCov(X = X, ix = ix, Y = Y, iy = iy)
> all.equal(ls1[-grep("Table", names(ls1))],
+           ls3[-grep("Table", names(ls3))])

[1] TRUE

> ### works also with factors
> ls3 <- LinStatExpCov(X = X, ix = factor(ix), Y = Y, iy = factor(iy))
> all.equal(ls1[-grep("Table", names(ls1))],
+           ls3[-grep("Table", names(ls3))])
```

```
[1] TRUE
```

Similar to the one-dimensional case, we can also omit the X matrix here

```
> ls4 <- LinStatExpCov(ix = ix, Y = Y, iy = iy)
> all.equal(ls3[-grep("Xfactor", names(ls3))],
+           ls4[-grep("Xfactor", names(ls4))])
```

```
[1] TRUE
```

It is important to note that all computations are based on the tabulations

```
> ls3$Table
```

```
, , 1

     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12]
[1,]    0    0    0    0    0    0    0    0    0     0     0     0
[2,]    0    0    4    4    1    2    3    0    1     2     3     0
[3,]    0    2    2    1    2    2    5    0    1     1     3     1
[4,]    0    1    1    4    0    1    5    2    0     2     3     1
[5,]    0    0    2    2    4    2    2    1    3     2     1     1
[6,]    0    1    3    1    1    1    2    2    2     6     1     0
```

```
> xtabs(~ ix + iy)
```

```
   iy
ix   1  2  3  4  5  6  7  8  9 10 11
  1  0  4  4  1  2  3  0  1  2  3  0
  2  2  2  1  2  2  5  0  1  1  3  1
  3  1  1  4  0  1  5  2  0  2  3  1
  4  0  2  2  4  2  2  1  3  2  1  1
  5  1  3  1  1  1  2  2  2  6  1  0
```

where the former would record missing values in the first row / column.

2.1.3 Methods and Tests

Objects of class LinStatExpCov returned by LinStatExpCov() contain the symmetric covariance matrix as a vector of the lower triangular elements. The vcov method allows to extract the full covariance matrix from such an object.

⟨ *vcov LinStatExpCov* 10 ⟩ ≡

```
vcov.LinStatExpCov <-
function(object, ...)
{
    if (object$varonly)
        stop("cannot extract covariance matrix")
    drop(.Call(R_unpack_sym, object$Covariance, NULL, 0L))
}
◇
```

Fragment referenced in 3a.
Uses: R_unpack_sym 149.

```
> ls1$Covariance
```

```
 [1]  1.3572364 -0.3393091 -0.3393091 -0.3393091 -0.3393091  1.3572364
 [7] -0.3393091 -0.3393091 -0.3393091  1.3572364 -0.3393091 -0.3393091
[13]  1.3572364 -0.3393091  1.3572364
```

```
> vcov(ls1)
```

```
           [,1]       [,2]       [,3]       [,4]       [,5]
[1,]  1.3572364 -0.3393091 -0.3393091 -0.3393091 -0.3393091
[2,] -0.3393091  1.3572364 -0.3393091 -0.3393091 -0.3393091
[3,] -0.3393091 -0.3393091  1.3572364 -0.3393091 -0.3393091
[4,] -0.3393091 -0.3393091 -0.3393091  1.3572364 -0.3393091
[5,] -0.3393091 -0.3393091 -0.3393091 -0.3393091  1.3572364
```

The most important task is, however, to compute test statistics and p-values. `doTest()` allows to compute the statistics $c_{\max}$ (taking `alternative` into account) and c_{quad} along with the corresponding p-values. If `nresample = 0` was used in the call to `LinStatExpCov()`, p-values are obtained from the limiting asymptotic distribution whenever such a thing is available at reasonable costs. Otherwise, the permutation p-value is returned (along with the permuted test statistics when `PermutedStatistics` is `TRUE`). The p-values (`lower = FALSE`) or $(1-p)$-values (`lower = TRUE`) can be computed on the log-scale.

⟨ *doTest Prototype* 11 ⟩ ≡

```
(object, teststat = c("maximum", "quadratic", "scalar"),
 alternative = c("two.sided", "less", "greater"), pvalue = TRUE,
 lower = FALSE, log = FALSE, PermutedStatistics = FALSE,
 minbucket = 10L, ordered = TRUE, maxselect = object$Xfactor,
 pargs = GenzBretz())◇
```

Fragment referenced in 12, 19.

⟨ *doTest* 12 ⟩ ≡

```
### note: lower = FALSE => p-value; lower = TRUE => 1 - p-value
doTest <-
function⟨ doTest Prototype 11 ⟩
{
    teststat <- match.arg(teststat, choices = c("maximum", "quadratic", "scalar"))
    if (!any(teststat == c("maximum", "quadratic", "scalar")))
        stop("incorrect teststat")
    alternative <- alternative[1]
    if (!any(alternative == c("two.sided", "less", "greater")))
        stop("incorrect alternative")

    if (maxselect)
        stopifnot(object$Xfactor)

    if (teststat == "quadratic" || maxselect) {
        if (alternative != "two.sided")
            stop("incorrect alternative")
    }

    test <- which(c("maximum", "quadratic", "scalar") == teststat)
    if (test == 3) {
        if (length(object$LinearStatistic) != 1)
            stop("scalar test statistic not applicable")
        test <- 1L # scalar is maximum internally
    }
    alt <- which(c("two.sided", "less", "greater") == alternative)

    if (!pvalue & (NCOL(object$PermutedLinearStatistic) > 0))
        object$PermutedLinearStatistic <- matrix(NA_real_, nrow = 0, ncol = 0)

    if (!maxselect) {
        if (teststat == "quadratic") {
            ret <- .Call(R_QuadraticTest, object, as.integer(pvalue), as.integer(lower),
                         as.integer(log), as.integer(PermutedStatistics))
        } else {
            ret <- .Call(R_MaximumTest, object, as.integer(alt), as.integer(pvalue),
                         as.integer(lower), as.integer(log), as.integer(PermutedStatistics),
                         as.integer(pargs$maxpts), as.double(pargs$releps),
                         as.double(pargs$abseps))
            if (teststat == "scalar") {
                var <- if (object$varonly) object$Variance else object$Covariance
                ret$TestStatistic <- object$LinearStatistic - object$Expectation
                ret$TestStatistic <-
                    if (var > object$tol) ret$TestStatistic / sqrt(var) else NaN
            }
        }
    } else {
        ret <- .Call(R_MaximallySelectedTest, object, as.integer(ordered), as.integer(test),
                     as.integer(minbucket), as.integer(lower), as.integer(log))
    }
    if (!PermutedStatistics) ret$PermutedStatistics <- NULL
    ret
}
◇
```

Fragment referenced in 3a.
Uses: `NCOL` 139c.

```
> ### c_max test statistic
> ### no p-value
> doTest(ls1, teststat = "maximum", pvalue = FALSE)

$TestStatistic
[1] 0.8411982

$p.value
[1] NA

> ### p-value
> doTest(ls1, teststat = "maximum")

$TestStatistic
[1] 0.8411982

$p.value
[1] 0.8852087

> ### log(p)-value
> doTest(ls1, teststat = "maximum", log = TRUE)

$TestStatistic
[1] 0.8411982

$p.value
[1] 0.108822

> ### (1-p)-value
> doTest(ls1, teststat = "maximum", lower = TRUE)

$TestStatistic
[1] 0.8411982

$p.value
[1] 0.1150168

> ### log(1 - p)-value
> doTest(ls1, teststat = "maximum", lower = TRUE, log = TRUE)

$TestStatistic
[1] 0.8411982

$p.value
[1] -2.164164

> ### quadratic
> doTest(ls1, teststat = "quadratic")

$TestStatistic
[1] 1.077484

$p.value
[1] 0.897828
```

Sometimes we are interested in contrasts of linear statistics and their corresponding properties. Examples include linear-by-linear association tests, where we assign numeric scores to each level of a factor. To implement this, we implement `lmult` so that we can then left-multiply a matrix to an object of class `LinStatExpCov`.

⟨ *Contrasts* 14 ⟩ ≡

```
lmult <-
function(x, object)
{
    stopifnot(!object$varonly)
    stopifnot(is.numeric(x))
    if (is.vector(x)) x <- matrix(x, nrow = 1)
    P <- object$dimension[1]
    stopifnot(ncol(x) == P)
    Q <- object$dimension[2]
    ret <- object
    xLS <- x %*% matrix(object$LinearStatistic, nrow = P)
    xExp <- x %*% matrix(object$Expectation, nrow = P)
    xExpX <- x %*% matrix(object$ExpectationX, nrow = P)
    if (Q == 1) {
        xCov <- tcrossprod(x %*% vcov(object), x)
    } else {
        zmat <- matrix(0, nrow = P * Q, ncol = nrow(x))
        mat <- rbind(t(x), zmat)
        mat <- mat[rep(1:nrow(mat), Q - 1),,drop = FALSE]
        mat <- rbind(mat, t(x))
        mat <- matrix(mat, ncol = Q * nrow(x))
        mat <- t(mat)
        xCov <- tcrossprod(mat %*% vcov(object), mat)
    }
    if (!is.matrix(xCov)) xCov <- matrix(xCov)
    if (length(object$PermutedLinearStatistic) > 0) {
        xPS <- apply(object$PermutedLinearStatistic, 2, function(y)
                     as.vector(x %*% matrix(y, nrow = P)))
        if (!is.matrix(xPS)) xPS <- matrix(xPS, nrow = 1)
        ret$PermutedLinearStatistic <- xPS
    }
    ret$LinearStatistic <- as.vector(xLS)
    ret$Expectation <- as.vector(xExp)
    ret$ExpectationX <- as.vector(xExpX)
    ret$Covariance <- as.vector(xCov[lower.tri(xCov, diag = TRUE)])
    ret$Variance <- diag(xCov)
    ret$dimension <- c(NROW(x), Q)
    ret$Xfactor <- FALSE
    if (length(object$StandardisedPermutedLinearStatistic) > 0)
        ret$StandardisedPermutedLinearStatistic <-
            .Call(R_StandardisePermutedLinearStatistic, ret)
    ret
}
◇
```

Fragment referenced in 3a.
Uses: `NROW` 139b, `P` 25a, `Q` 25e, `x` 24d, 25bc, `y` 25d, 26ab.

Here is an example for a linear-by-linear association test.

```
> set.seed(29)
```

```
> ls1d <- LinStatExpCov(X = model.matrix(~ x - 1), Y = matrix(y, ncol = 1),
+                       nresample = 10, standardise = TRUE)
> set.seed(29)
> ls1s <- LinStatExpCov(X = as.double(1:5)[x], Y = matrix(y, ncol = 1),
+                       nresample = 10, standardise = TRUE)
> ls1c <- lmult(c(1:5), ls1d)
> stopifnot(isequal(ls1c, ls1s))
> set.seed(29)
> ls1d <- LinStatExpCov(X = model.matrix(~ x - 1), Y = matrix(c(y, y), ncol = 2),
+                       nresample = 10, standardise = TRUE)
> set.seed(29)
> ls1s <- LinStatExpCov(X = as.double(1:5)[x], Y = matrix(c(y, y), ncol = 2),
+                       nresample = 10, standardise = TRUE)
> ls1c <- lmult(c(1:5), ls1d)
> stopifnot(isequal(ls1c, ls1s))
```

2.1.4 Tabulations

The tabulation of `ix` and `iy` can be computed without necessarily computing the corresponding linear statistics via `ctabs()`.

⟨ *ctabs Prototype* 15 ⟩ ≡

```
    (ix, iy = integer(0), block = integer(0), weights = integer(0),
     subset = integer(0), checkNAs = TRUE)◇
```

Fragment referenced in 16, 20.
Uses: block 28bd, subset 27be, 28a, weights 26c.

"ctabs.R" 16≡

```
⟨ R Header 166a ⟩
ctabs <-
function⟨ ctabs Prototype 15 ⟩
{
    stopifnot(is.integer(ix) || is.factor(ix))
    N <- length(ix)

    ⟨ Check ix 9a ⟩

    if (length(iy) > 0) {
        stopifnot(length(iy) == N)
        stopifnot(is.integer(iy) || is.factor(iy))
        ⟨ Check iy 9b ⟩
    }

    ⟨ Check weights, subset, block 5a ⟩

    if (length(iy) == 0 && length(block) == 0)
        return(.Call(R_OneTableSums, ix, weights, subset))
    if (length(block) == 0)
        return(.Call(R_TwoTableSums, ix, iy, weights, subset))
    if (length(iy) == 0)
        return(.Call(R_TwoTableSums, ix, block, weights, subset)[,-1,drop = FALSE])
    return(.Call(R_ThreeTableSums, ix, iy, block, weights, subset))
}
◇
```

Uses: `block` 28bd, `N` 24bc, `R_OneTableSums` 118a, `R_ThreeTableSums` 127b, `R_TwoTableSums` 122b, `subset` 27be, 28a, `weights` 26c, `weights`, 26de.

```
> t1 <- ctabs(ix = ix, iy = iy)
> t2 <- xtabs(~ ix + iy)
> max(abs(t1[-1, -1] - t2))

[1] 0
```

2.2 Manual Pages

"LinStatExpCov.Rd" 18≡

```
\name{LinStatExpCov}
\alias{LinStatExpCov}
\alias{lmult}
\title{
  Linear Statistics with Expectation and Covariance
}
\description{
  Strasser-Weber type linear statistics and their expectation
  and covariance under the independence hypothesis
}
\usage{
LinStatExpCov⟨ LinStatExpCov Prototype 3b ⟩
lmult(x, object)
}
\arguments{
  \item{X}{numeric matrix of transformations.}
  \item{Y}{numeric matrix of influence functions.}
  \item{ix}{an optional integer vector expanding \code{X}.}
  \item{iy}{an optional integer vector expanding \code{Y}.}
  \item{weights}{an optional integer vector of non-negative case weights.}
  \item{subset}{an optional integer vector defining a subset of observations.}
  \item{block}{an optional factor defining independent blocks of observations.}
  \item{checkNAs}{a logical for switching off missing value checks.  This
    included switching off checks for suitable values of \code{subset}.
    Use at your own risk.}
  \item{varonly}{a logical asking for variances only.}
  \item{nresample}{an integer defining the number of permuted statistics to draw.}
  \item{standardise}{a logical asking to standardise the permuted statistics.}
  \item{tol}{tolerance for zero variances.}
  \item{x}{a contrast matrix to be left-multiplied in case \code{X} was a factor.}
  \item{object}{an object of class \code{LinStatExpCov}.}
}
\details{
  The function, after minimal preprocessing, calls the underlying C code
  and computes the linear statistic, its expectation and covariance and,
  optionally, \code{nresample} samples from its permutation distribution.

  When both \code{ix} and \code{iy} are missing, the number of rows of
  \code{X} and \code{Y} is the same, ie the number of observations.

  When \code{X} is missing and \code{ix} a factor, the code proceeds as
  if \code{X} were a dummy matrix of \code{ix} without explicitly
  computing this matrix.

  Both \code{ix} and \code{iy} being present means the code treats them
  as subsetting vectors for \code{X} and \code{Y}.  Note that \code{ix = 0}
  or \code{iy = 0} means that the corresponding observation is missing
  and the first row or \code{X} and \code{Y} must be zero.

  \code{lmult} allows left-multiplication of a contrast matrix when \code{X}
  was (equivalent to) a factor.
}
\value{
  A list.
}
\references{
  Strasser, H. and Weber, C.  (1999).  On the asymptotic theory of permutation
  statistics.  \emph{Mathematical Methods of Statistics} \bold{8}(2), 220--250.
}
\examples{
wilcox.test(Ozone ~ Month, data = airquality, subset = Month \%in\% c(5, 8))

aq <- subset(airquality, Month \%in\% c(5, 8))
X <- as.double(aq$Month == 5)
Y <- as.double(rank(aq$Ozone))
doTest(LinStatExpCov(X, Y))
}
```

"doTest.Rd" 19≡

```
\name{doTest}
\alias{doTest}
\title{
  Permutation Test
}
\description{
  Perform permutation test for a linear statistic
}
\usage{
doTest⟨ doTest Prototype 11 ⟩
}
\arguments{
  \item{object}{an object returned by \code{\link{LinStatExpCov}}.}
  \item{teststat}{type of test statistic to use.}
  \item{alternative}{alternative for scalar or maximum-type statistics.}
  \item{pvalue}{a logical indicating if a p-value shall be computed.}
  \item{lower}{a logical indicating if a p-value (\code{lower} is \code{FALSE})
    or 1 - p-value (\code{lower} is \code{TRUE}) shall be returned.}
  \item{log}{a logical, if \code{TRUE} probabilities are log-probabilities.}
  \item{PermutedStatistics}{a logical, return permuted test statistics.}
  \item{minbucket}{minimum weight in either of two groups for maximally selected
    statistics.}
  \item{ordered}{a logical, if \code{TRUE} maximally selected statistics assume
    that the cutpoints are ordered.}
  \item{maxselect}{a logical, if \code{TRUE} maximally selected statistics are
    computed.  This requires that \code{X} was an implicitly defined design
    matrix in \code{\link{LinStatExpCov}}.}
  \item{pargs}{arguments as in \code{\link[mvtnorm:algorithms]{GenzBretz}}.}
}
\details{
  Computes a test statistic, a corresponding p-value and, optionally, cutpoints
  for maximally selected statistics.
}
\value{
  A list.
}
\keyword{htest}
◇
```

"ctabs.Rd" 20≡

```
\name{ctabs}
\alias{ctabs}
\title{
  Cross Tabulation
}
\description{
  Efficient weighted cross tabulation of two factors and a block
}
\usage{
ctabs⟨ ctabs Prototype 15 ⟩
}
\arguments{
  \item{ix}{a integer of positive values with zero indicating a missing.}
  \item{iy}{an optional integer of positive values with zero indicating a
    missing.}
  \item{block}{an optional blocking factor without missings.}
  \item{weights}{an optional vector of weights, integer or double.}
  \item{subset}{an optional integer vector indicating a subset.}
  \item{checkNAs}{a logical for switching off missing value checks.}
}
\details{
  A faster version of \code{xtabs(weights ~ ix + iy + block, subset)}.
}
\value{
  If \code{block} is present, a three-way table. Otherwise,
  a one- or two-dimensional table.
}
\examples{
ctabs(ix = 1:5, iy = 1:5, weights = 1:5 / 5)
}
\keyword{univar}
◇
```

Uses: `block` 28bd, `subset` 27be, 28a, `weights` 26c, `weights,` 26de.

Chapter 3

C Code

The main motivation to implement the **libcoin** package comes from the demand to compute high-dimensional linear statistics (with large P and Q) and the corresponding test statistics very often, either for sampling from the permutation null distribution H_0 or for different subsets of the data. Especially the latter task can be performed *without* actually subsetting the data via the `subset` argument very efficiently (in terms of memory consumption and, depending on the circumstances, speed).

We start with the definition of some macros and global variables in the header files.

3.1 Header and Source Files

`"libcoin_internal.h"` 21a≡

```
⟨ C Header 166b ⟩
⟨ R Includes 21b ⟩
⟨ C Macros 22a ⟩
⟨ C Global Variables 22b ⟩
◇
```

These includes provide some R infrastructure at C level.

⟨ *R Includes* 21b ⟩ ≡

```
#include <R.h>
#include <Rinternals.h>
#include <Rmath.h>
#include <Rdefines.h>
#include <R_ext/stats_package.h> /* for S_rcont2 */
#include <Rversion.h>           // for R_VERSION
#include <R_ext/Lapack.h> /* for dspev */
◇
```

Fragment referenced in 21a.

We need three macros: `S` computes the element Σ_{ij} of a symmetric $n \times n$ matrix when only the lower triangular elements are stored. `LE` implements $\leq$ with some tolerance, `GE` implements $\geq$.

⟨ *C Macros* 22a ⟩ ≡

```
#define S(i, j, n) ((i) >= (j) ? (n) * (j) + (i) - (j) * ((j) + 1) / 2 : (n) * (i) + (j) -
(i) * ((i) + 1) / 2)
#define LE(x, y, tol) ((x) < (y)) || (fabs((x) - (y)) < (tol))
#define GE(x, y, tol) ((x) > (y)) || (fabs((x) - (y)) < (tol))
◇
```

Fragment referenced in 21a.
Defines: GE 55, 57, LE 57, S 37b, 38b, 47, 48, 60b, 61b, 62b, 65, 67a, 71, 72a, 76a, 80b, 93a, 105, 144, 145a, 147, 153b.
Uses: x 24d, 25bc, y 25d, 26ab.

⟨ *C Global Variables* 22b ⟩ ≡

```
#define ALTERNATIVE_twosided                    1
#define ALTERNATIVE_less                        2
#define ALTERNATIVE_greater                     3

#define TESTSTAT_maximum                        1
#define TESTSTAT_quadratic                      2

#define LinearStatistic_SLOT                    0
#define Expectation_SLOT                        1
#define Covariance_SLOT                         2
#define Variance_SLOT                           3
#define ExpectationX_SLOT                       4
#define varonly_SLOT                            5
#define dim_SLOT                                6
#define ExpectationInfluence_SLOT               7
#define CovarianceInfluence_SLOT                8
#define VarianceInfluence_SLOT                  9
#define Xfactor_SLOT                            10
#define tol_SLOT                                11
#define PermutedLinearStatistic_SLOT            12
#define StandardisedPermutedLinearStatistic_SLOT        13
#define TableBlock_SLOT                         14
#define Sumweights_SLOT                         15
#define Table_SLOT                              16

#define DoSymmetric                             1
#define DoCenter                                1
#define DoVarOnly                               1
#define Power1                                  1
#define Power2                                  2
#define Offset0                                 0
◇
```

Fragment referenced in 21a.
Defines: CovarianceInfluence_SLOT 155a, 158b, 159, Covariance_SLOT 153b, 154a, 158b, 159, dim_SLOT 151c, 152a, 158b, 159, DoCenter 81d, 86a, 88a, 90, 93a, 100a, 113b, DoSymmetric 81d, 88a, 93a, DoVarOnly 37bc, 38a, 47, ExpectationInfluence_SLOT 154c, 158b, 159, ExpectationX_SLOT 154b, 158b, 159, Expectation_SLOT 153a, 158b, 159, LinearStatistic_SLOT 152d, 158b, 159, Offset0 35b, 36a, 40, 44, 46c, 47, 85b, 87a, 89a, 92a, 95b, 100a, 109b, 113b, 118a, 122b, 127b, 132b, 136a, PermutedLinearStatistic_SLOT 157bc, 158b, 159, Power1 86a, 90, 113b, Power2 88a, 93a, StandardisedPermutedLinearStatistic_SLOT 158b, 159, Sumweights_SLOT 156a, 157a, 158b, 159, 160b, TableBlock_SLOT 36a, 155c, 157a, 158b, 159, 160b, Table_SLOT 156bc, 158b, 159, 161, tol_SLOT 157d, 158b, 159, VarianceInfluence_SLOT 155b, 158b, 159, Variance_SLOT 153b, 158b, 159, varonly_SLOT 152b, 158b, 159, Xfactor_SLOT 152c, 158b, 159.

The corresponding header file contains definitions of functions that can be called via `.Call()` from the **libcoin** package. In addition, packages linking to **libcoin** can access these function at C level (at your own risk, of course!).

"libcoin.h" 23a≡

```
⟨ C Header 166b ⟩
#include "libcoin_internal.h"
⟨ Function Prototypes 23b ⟩
◇
```

⟨ *Function Prototypes* 23b ⟩ ≡

```
extern ⟨ R_ExpectationCovarianceStatistic Prototype 32b ⟩;
extern ⟨ R_PermutedLinearStatistic Prototype 38c ⟩;
extern ⟨ R_StandardisePermutedLinearStatistic Prototype 41c ⟩;
extern ⟨ R_ExpectationCovarianceStatistic_2d Prototype 43a ⟩;
extern ⟨ R_PermutedLinearStatistic_2d Prototype 50a ⟩;
extern ⟨ R_QuadraticTest Prototype 54b ⟩;
extern ⟨ R_MaximumTest Prototype 56b ⟩;
extern ⟨ R_MaximallySelectedTest Prototype 58 ⟩;
extern ⟨ R_ExpectationInfluence Prototype 85a ⟩;
extern ⟨ R_CovarianceInfluence Prototype 86b ⟩;
extern ⟨ R_ExpectationX Prototype 88b ⟩;
extern ⟨ R_CovarianceX Prototype 91 ⟩;
extern ⟨ R_Sums Prototype 95a ⟩;
extern ⟨ R_KronSums Prototype 99 ⟩;
extern ⟨ R_KronSums_Permutation Prototype 109a ⟩;
extern ⟨ R_colSums Prototype 113a ⟩;
extern ⟨ R_OneTableSums Prototype 117b ⟩;
extern ⟨ R_TwoTableSums Prototype 122a ⟩;
extern ⟨ R_ThreeTableSums Prototype 127a ⟩;
extern ⟨ R_order_subset_wrt_block Prototype 132a ⟩;
extern ⟨ R_quadform Prototype 64b ⟩;
extern ⟨ R_kronecker Prototype 141c ⟩;
extern ⟨ R_MPinv_sym Prototype 146a ⟩;
extern ⟨ R_unpack_sym Prototype 148b ⟩;
extern ⟨ R_pack_sym Prototype 150b ⟩;
◇
```

Fragment referenced in 23a.

The C file `libcoin.c` contains all C functions and corresponding R interfaces.

"libcoin.c" 23c≡

```
⟨ C Header 166b ⟩
#include "libcoin_internal.h"
#include <R_ext/stats_stubs.h>  /* for S_rcont2 */
#include <mvtnormAPI.h>         /* for calling mvtnorm */
⟨ Function Definitions 24a ⟩
◇
```

⟨ *Function Definitions* 24a ⟩ ≡

```
⟨ MoreUtils 139a ⟩
⟨ Memory 151a ⟩
⟨ P-Values 67b ⟩
⟨ KronSums 98b ⟩
⟨ colSums 112c ⟩
⟨ SimpleSums 94c ⟩
⟨ Tables 117a ⟩
⟨ Utils 131b ⟩
⟨ LinearStatistics 81b ⟩
⟨ Permutations 136b ⟩
⟨ ExpectationCovariances 82a ⟩
⟨ Test Statistics 60a ⟩
⟨ User Interface 31a ⟩
⟨ 2d User Interface 42b ⟩
⟨ Tests 53b ⟩
◇
```

Fragment referenced in 23c.

3.2 Variables

N is the number of observations

⟨ *R N Input* 24b ⟩ ≡

```
SEXP N,
◇
```

Fragment referenced in 95a.
Defines: `N` 5ab, 6, 8, 16, 24c, 35ab, 36ab, 37abc, 38a, 40, 44, 70, 81d, 85b, 86a, 87a, 88a, 89a, 90, 92a, 93ab, 94a, 95b, 96a, 98a, 100a, 102, 103a, 105, 108, 109b, 110a, 111b, 112b, 113b, 114a, 116b, 118a, 119a, 122b, 123b, 127b, 128b, 132b, 133b, 134ab, 135a, 136a, 145a.

which at C level is represented as `R_xlen_t` to allow for $N >$ `INT_MAX`

⟨ *C integer N Input* 24c ⟩ ≡

```
R_xlen_t N
◇
```

Fragment referenced in 25bc, 34, 40, 44, 81c, 85bc, 87ab, 89ab, 92ab, 95c, 96b, 97abc, 100a, 101b, 109bc, 113b, 118a, 122b, 127b, 132b, 133a, 134ab, 135b.
Defines: `N` 5ab, 6, 8, 16, 24b, 35ab, 36ab, 37abc, 38a, 40, 44, 70, 81d, 85b, 86a, 87a, 88a, 89a, 90, 92a, 93ab, 94a, 95b, 96a, 98a, 100a, 102, 103a, 105, 108, 109b, 110a, 111b, 112b, 113b, 114a, 116b, 118a, 119a, 122b, 123b, 127b, 128b, 132b, 133b, 134ab, 135a, 136a, 145a.

The regressors $\mathbf{x}_i, i = 1, \dots, N$

⟨ *R x Input* 24d ⟩ ≡

```
SEXP x,
◇
```

Fragment referenced in 31b, 42c, 50a, 81c, 88b, 89b, 91, 92b, 99, 101b, 109ac, 113a, 117b, 122a, 127a.
Defines: `x` 8, 14, 18, 22a, 25bc, 32ac, 33, 35ab, 37ac, 38ad, 40, 43b, 44, 45ab, 46c, 47, 50b, 51, 81d, 89a, 90, 92a, 93a, 100a, 101a, 102, 103a, 105, 108, 109b, 110a, 111b, 112b, 113b, 114a, 116b, 118a, 119a, 121b, 122b, 123b, 126, 127b, 128b, 131a, 139bc, 140a, 145ab, 146ab, 147, 148ab, 149, 150abc.

are either represented as a real matrix with N rows and P columns

⟨ *C integer P Input* 25a ⟩ ≡

```
int P
```
◇

Fragment referenced in 25bc, 34, 81c, 82b, 83, 84, 89b, 92b, 101b, 109c, 160b, 161.
Defines: `P` 14, 32c, 33, 35ab, 36a, 37ac, 38ab, 40, 44, 45ab, 46c, 47, 48, 49, 51, 55, 56a, 57, 59, 73, 74, 75, 76a, 78, 79ab, 80ab, 81d, 82b, 83, 84, 88b, 89a, 90, 91, 92a, 93a, 99, 100a, 102, 103a, 105, 108, 109ab, 110a, 111b, 112b, 113b, 114a, 116b, 118a, 119a, 121b, 122b, 123b, 126, 127b, 128b, 131a, 140b, 141a, 145a, 158a, 159.

⟨ *C real x Input* 25b ⟩ ≡

```
double *x,
```
⟨ *C integer N Input* 24c ⟩,
⟨ *C integer P Input* 25a ⟩,
◇

Fragment referenced in 101c, 110b, 111a, 114b, 145a.
Defines: `x` 8, 14, 18, 22a, 24d, 25c, 32ac, 33, 35ab, 37ac, 38ad, 40, 43b, 44, 45ab, 46c, 47, 50b, 51, 81d, 89a, 90, 92a, 93a, 100a, 101a, 102, 103a, 105, 108, 109b, 110a, 111b, 112b, 113b, 114a, 116b, 118a, 119a, 121b, 122b, 123b, 126, 127b, 128b, 131a, 139bc, 140a, 145ab, 146ab, 147, 148ab, 149, 150abc.

or as a factor (an integer at C level) at P levels

⟨ *C integer x Input* 25c ⟩ ≡

```
int *x,
```
⟨ *C integer N Input* 24c ⟩,
⟨ *C integer P Input* 25a ⟩,
◇

Fragment referenced in 106a, 111c, 112a, 119b, 123c, 128c.
Defines: `x` 8, 14, 18, 22a, 24d, 25b, 32ac, 33, 35ab, 37ac, 38ad, 40, 43b, 44, 45ab, 46c, 47, 50b, 51, 81d, 89a, 90, 92a, 93a, 100a, 101a, 102, 103a, 105, 108, 109b, 110a, 111b, 112b, 113b, 114a, 116b, 118a, 119a, 121b, 122b, 123b, 126, 127b, 128b, 131a, 139bc, 140a, 145ab, 146ab, 147, 148ab, 149, 150abc.

The influence functions are also either a $N \times Q$ real matrix

⟨ *R y Input* 25d ⟩ ≡

```
SEXP y,
```
◇

Fragment referenced in 31b, 42c, 50a, 85ac, 86b, 87b, 99, 109a, 122a, 127a, 132a.
Defines: `y` 14, 18, 22a, 26ab, 32ac, 33, 35b, 37ab, 38d, 40, 43b, 44, 45ab, 46c, 47, 50b, 81d, 85b, 86a, 87a, 88a, 100a, 102, 103a, 105, 108, 109b, 110a, 111b, 112b, 122b, 123b, 126, 127b, 128b, 131a, 132b, 143, 144.

⟨ *C integer Q Input* 25e ⟩ ≡

```
int Q
```
◇

Fragment referenced in 26ab, 34, 82b, 83, 84, 85bc, 87ab, 100a, 109b, 160b, 161.
Defines: `Q` 14, 32c, 33, 35ab, 37abc, 38ab, 40, 44, 45ab, 46c, 47, 48, 49, 51, 55, 56a, 57, 73, 74, 75, 76abc, 78, 80ab, 81ad, 82b, 83, 84, 85b, 86a, 87a, 88a, 100a, 102, 103a, 105, 108, 109b, 110a, 111b, 112b, 122b, 123b, 126, 127b, 128b, 131a, 141a, 158a, 159, 160a.

⟨ *C real y Input* 26a ⟩ ≡

```
double *y,
⟨ C integer Q Input 25e ⟩,
◇
```

Fragment referenced in 81c, 101bc, 106a, 109c, 110b, 111ac, 112a.
Defines: `y` 14, 18, 22a, 25d, 26b, 32ac, 33, 35b, 37ab, 38d, 40, 43b, 44, 45ab, 46c, 47, 50b, 81d, 85b, 86a, 87a, 88a, 100a, 102, 103a, 105, 108, 109b, 110a, 111b, 112b, 122b, 123b, 126, 127b, 128b, 131a, 132b, 143, 144.

or a factor at Q levels

⟨ *C integer y Input* 26b ⟩ ≡

```
int *y,
⟨ C integer Q Input 25e ⟩,
◇
```

Fragment referenced in 123c, 128c.
Defines: `y` 14, 18, 22a, 25d, 26a, 32ac, 33, 35b, 37ab, 38d, 40, 43b, 44, 45ab, 46c, 47, 50b, 81d, 85b, 86a, 87a, 88a, 100a, 102, 103a, 105, 108, 109b, 110a, 111b, 112b, 122b, 123b, 126, 127b, 128b, 131a, 132b, 143, 144.

The weights $w_i, i = 1, \dots, N$

⟨ *R weights Input* 26c ⟩ ≡

```
SEXP weights
◇
```

Fragment referenced in 31b, 42c, 81c, 85ac, 86b, 87b, 88b, 89b, 91, 92b, 95ac, 99, 100b, 113ac, 117b, 118b, 122a, 123a, 127a, 128a, 132a, 135b.
Defines: `weights` 3b, 4, 5a, 6, 8, 15, 16, 18, 20, 26de, 32ac, 35b, 36b, 37abc, 38ad, 40, 43b, 44, 52a, 81d, 85b, 86a, 87a, 88a, 89a, 90, 92a, 93ab, 95b, 96a, 100a, 102, 103a, 113b, 114a, 118a, 119a, 122b, 123b, 127b, 128b, 132b, 136a.

can be constant one (`XLENGTH(weights) == 0` or `weights = integer(0)`) or integer-valued, with `HAS_WEIGHTS == 0` in the former case

⟨ *C integer weights Input* 26d ⟩ ≡

```
int *weights,
int HAS_WEIGHTS,
◇
```

Fragment referenced in 97ab, 104ab, 106c, 107a, 115bc, 120bc, 124c, 125a, 129c, 130a.
Defines: `HAS_WEIGHTS` 26e, 98a, 105, 108, 116b, 121b, 126, 131a, `weights,` 4, 6, 8, 16, 20, 26e, 32ac, 35b, 36b, 37abc, 38ad, 40, 43b, 44, 81d, 85b, 86a, 87a, 88a, 89a, 90, 92a, 93a, 95b, 100a, 113b, 118a, 122b, 127b, 132b, 136a.
Uses: `weights` 26c.

Weights larger than `INT_MAX` are stored as double

⟨ *C real weights Input* 26e ⟩ ≡

```
double *weights,
int HAS_WEIGHTS,
◇
```

Fragment referenced in 96b, 97c, 103b, 104c, 106b, 107b, 115a, 116a, 120a, 121a, 124b, 125b, 129b, 130b.
Defines: `HAS_WEIGHTS` 26d, 98a, 105, 108, 116b, 121b, 126, 131a, `weights,` 4, 6, 8, 16, 20, 26d, 32ac, 35b, 36b, 37abc, 38ad, 40, 43b, 44, 81d, 85b, 86a, 87a, 88a, 89a, 90, 92a, 93a, 95b, 100a, 113b, 118a, 122b, 127b, 132b, 136a.
Uses: `weights` 26c.

The sum of all weights is a double

⟨ *C sumweights Input* 27a ⟩ ≡

```
double sumweights
```
◇

Fragment referenced in 83, 84, 85c, 87b.
Defines: `sumweights` 34, 36ab, 37abc, 38a, 46bc, 47, 49, 51, 52b, 53a, 74, 75, 76b, 81a, 83, 84, 85b, 86a, 87a, 88a, 136a, 156a.

Subsets $\mathcal{A} \subseteq \{1, \dots, N\}$ are R style indices

⟨ *R subset Input* 27b ⟩ ≡

```
SEXP subset
```
◇

Fragment referenced in 31b, 42c, 81c, 85ac, 86b, 87b, 88b, 89b, 91, 92b, 95ac, 99, 100b, 109ac, 113ac, 117b, 118b, 122a, 123a, 127a, 128a, 132a, 133a, 135ab.
Defines: `subset` 3b, 4, 5ab, 6, 8, 15, 16, 18, 20, 27e, 28a, 32ac, 34, 35b, 36ab, 38d, 40, 43b, 44, 46c, 47, 81d, 85b, 86a, 87a, 88a, 89a, 90, 92a, 93ab, 94b, 95b, 96a, 100a, 102, 103a, 109b, 110a, 111b, 112b, 113b, 114a, 118a, 119a, 122b, 123b, 127b, 128b, 132b, 133b, 135a, 136a, 137ab, 138ab.

are either not existent (`XLENGTH(subset) == 0`) or of length

⟨ *C integer Nsubset Input* 27c ⟩ ≡

```
R_xlen_t Nsubset
```
◇

Fragment referenced in 27d, 40, 44, 85b, 87a, 89a, 92a, 95b, 100a, 109b, 113b, 118a, 122b, 127b, 137ab, 138b.
Defines: `Nsubset` 36b, 40, 44, 81d, 85b, 86a, 87a, 88a, 89a, 90, 92a, 93ab, 94ab, 95b, 96a, 98a, 100a, 102, 103a, 109b, 110a, 111b, 112b, 113b, 114a, 118a, 119a, 122b, 123b, 127b, 128b, 137ab, 138b.

Optionally, one can specify a subset of the subset via

⟨ *C subset range Input* 27d ⟩ ≡

```
R_xlen_t offset,
⟨ C integer Nsubset Input 27c ⟩
```
◇

Fragment referenced in 27e, 28a, 81c, 85c, 87b, 89b, 92b, 95c, 100b, 109c, 113c, 118b, 123a, 128a.
Defines: `offset` 34, 36b, 37abc, 38a, 81d, 86a, 88a, 90, 93ab, 96a, 102, 103a, 110a, 111b, 112b, 114a, 119a, 123b, 128b.

where `offset` is a C style index for `subset`.
Subsets are stored either as integer

⟨ *C integer subset Input* 27e ⟩ ≡

```
int *subset,
⟨ C subset range Input 27d ⟩
```
◇

Fragment referenced in 97bc, 104bc, 107ab, 111a, 112a, 115c, 116a, 120c, 121a, 125ab, 130ab.
Defines: `subset` 3b, 4, 5ab, 6, 8, 15, 16, 18, 20, 27b, 28a, 32ac, 34, 35b, 36ab, 38d, 40, 43b, 44, 46c, 47, 81d, 85b, 86a, 87a, 88a, 89a, 90, 92a, 93ab, 94b, 95b, 96a, 100a, 102, 103a, 109b, 110a, 111b, 112b, 113b, 114a, 118a, 119a, 122b, 123b, 127b, 128b, 132b, 133b, 135a, 136a, 137ab, 138ab.

or double (to allow for indices larger than `INT_MAX`)

⟨ *C real subset Input* 28a ⟩ ≡

```
double *subset,
⟨ C subset range Input 27d ⟩
◇
```

Fragment referenced in 96b, 97a, 103b, 104a, 106bc, 110b, 111c, 115ab, 120ab, 124bc, 129bc.
Defines: `subset` 3b, 4, 5ab, 6, 8, 15, 16, 18, 20, 27be, 32ac, 34, 35b, 36ab, 38d, 40, 43b, 44, 46c, 47, 81d, 85b, 86a, 87a, 88a, 89a, 90, 92a, 93ab, 94b, 95b, 96a, 100a, 102, 103a, 109b, 110a, 111b, 112b, 113b, 114a, 118a, 119a, 122b, 123b, 127b, 128b, 132b, 133b, 135a, 136a, 137ab, 138ab.

Blocks $\text{block}_i, i = 1, \dots, N$

⟨ *R block Input* 28b ⟩ ≡

```
SEXP block
◇
```

Fragment referenced in 31b, 42c, 50a, 127a, 132a, 133a, 134b, 135a.
Defines: `block` 3b, 4, 5a, 6, 8, 15, 16, 18, 20, 28d, 32ac, 33, 36ab, 38d, 40, 43b, 44, 45a, 50b, 127b, 128b, 131a, 132b, 133b, 134b, 135a, 155c.

at B levels

⟨ *C integer B Input* 28c ⟩ ≡

```
int B
◇
```

Fragment referenced in 28d, 34, 160b, 161.
Defines: `B` 32c, 33, 34, 35a, 36a, 40, 44, 45a, 46a, 48, 49, 51, 52b, 73, 74, 78, 127b, 128b, 131a, 141abc, 142, 143, 144, 158a, 159, 160b, 161.

are stored as a factor

⟨ *C integer block Input* 28d ⟩ ≡

```
int *block,
⟨ C integer B Input 28c ⟩,
◇
```

Fragment referenced in 128c.
Defines: `block` 3b, 4, 5a, 6, 8, 15, 16, 18, 20, 28b, 32ac, 33, 36ab, 38d, 40, 43b, 44, 45a, 50b, 127b, 128b, 131a, 132b, 133b, 134b, 135a, 155c.

The tabulation of block (potentially in subsets) is

⟨ *R blockTable Input* 28e ⟩ ≡

```
SEXP blockTable
◇
```

Fragment referenced in 133a, 134b, 135a.
Defines: `blockTable` 40, 132b, 133b, 134b, 135a.

where the table is of length $B + 1$ and the first element counts the number of missing values (although these are NOT allowed in block).

3.2.1 Example Data and Code

We start with setting-up some toy data sets to be used as test bed. The data over both the 1d and the 2d case, including weights, subsets and blocks.

```
> N <- 20L
> P <- 3L
> Lx <- 10L
> Ly <- 5L
> Q <- 4L
> B <- 2L
> iX2d <- rbind(0, matrix(runif(Lx * P), nrow = Lx))
> ix <- sample(1:Lx, size = N, replace = TRUE)
> levels(ix) <- 1:Lx
> ixf <- factor(ix, levels = 1:Lx, labels = 1:Lx)
> x <- iX2d[ix + 1,]
> Xfactor <- diag(Lx)[ix,]
> iY2d <- rbind(0, matrix(runif(Ly * Q), nrow = Ly))
> iy <- sample(1:Ly, size = N, replace = TRUE)
> levels(iy) <- 1:Ly
> iyf <- factor(iy, levels = 1:Ly, labels = 1:Ly)
> y <- iY2d[iy + 1,]
> weights <- sample(0:5, size = N, replace = TRUE)
> block <- sample(gl(B, ceiling(N / B))[1:N])
> subset <- sort(sample(1:N, floor(N * 1.5), replace = TRUE))
> subsety <- sample(1:N, floor(N * 1.5), replace = TRUE)
> r1 <- rep(1:ncol(x), ncol(y))
> r1Xfactor <- rep(1:ncol(Xfactor), ncol(y))
> r2 <- rep(1:ncol(y), each = ncol(x))
> r2Xfactor <- rep(1:ncol(y), each = ncol(Xfactor))
```

As a benchmark, we implement linear statistics, their expectation and covariance, taking weights, subsets and blocks into account, at R level. In a sense, the core of the **libcoin** package is “just” a less memory-hungry and sometimes faster version of this simple function.

```
> LECV <- function(X, Y, weights = integer(0), subset = integer(0), block = integer(0)) {
+
+     if (length(weights) == 0) weights <- rep(1, NROW(X))
+     if (length(subset) == 0) subset <- 1:NROW(X)
+     idx <- rep(subset, weights[subset])
+     X <- X[idx,,drop = FALSE]
+     Y <- Y[idx,,drop = FALSE]
+     sumweights <- length(idx)
+
+     if (length(block) == 0) {
+         ExpX <- colSums(X)
+         ExpY <- colSums(Y) / sumweights
+         yc <- t(t(Y) - ExpY)
+         CovY <- crossprod(yc) / sumweights
+         CovX <- crossprod(X)
+         Exp <- kronecker(ExpY, ExpX)
+         Cov <- sumweights / (sumweights - 1) * kronecker(CovY, CovX) -
+                1 / (sumweights - 1) * kronecker(CovY, tcrossprod(ExpX))
+
+         ret <- list(LinearStatistic = as.vector(crossprod(X, Y)),
```

```
+                       Expectation = as.vector(Exp),
+                       Covariance = Cov,
+                       Variance = diag(Cov))
+    } else {
+         block <- block[idx]
+         ret <- list(LinearStatistic = 0, Expectation = 0, Covariance = 0, Variance = 0)
+         for (b in levels(block)) {
+             tmp <- LECV(X = X, Y = Y, subset = which(block == b))
+             for (l in names(ret)) ret[[l]] <- ret[[l]] + tmp[[l]]
+         }
+    }
+    return(ret)
+ }
```

```
> cmpr <- function(ret1, ret2) {
+     if (inherits(ret1, "LinStatExpCov")) {
+         if (!ret1$varonly)
+             ret1$Covariance <- vcov(ret1)
+     }
+     ret1 <- ret1[!sapply(ret1, is.null)]
+     ret2 <- ret2[!sapply(ret2, is.null)]
+     nm1 <- names(ret1)
+     nm2 <- names(ret2)
+     nm <- c(nm1, nm2)
+     nm <- names(table(nm))[table(nm) == 2]
+     isequal(ret1[nm], ret2[nm])
+ }
```

We now compute the linear statistic along with corresponding expectation, variance and covariance for later reuse.

```
> LECVxyws <- LinStatExpCov(x, y, weights = weights, subset = subset)
> LEVxyws <- LinStatExpCov(x, y, weights = weights, subset = subset, varonly = TRUE)
```

The following tests compare the high-level R implementation (function `LECV()`) with the 1d and 2d C level implementations in the two sitations with and without specification of `X` (ie, the dummy matrix in the latter case).

```
> ### with X given
> testit <- function(...) {
+     a <- LinStatExpCov(x, y, ...)
+     b <- LECV(x, y, ...)
+     d <- LinStatExpCov(X = iX2d, ix = ix, Y = iY2d, iy = iy, ...)
+     return(cmpr(a, b) && cmpr(d, b))
+ }
> stopifnot(
+     testit() && testit(weights = weights) &&
+     testit(subset = subset) && testit(weights = weights, subset = subset) &&
+     testit(block = block) && testit(weights = weights, block = block) &&
+     testit(subset = subset, block = block) &&
+     testit(weights = weights, subset = subset, block = block))
> ### without dummy matrix X
> testit <- function(...) {
+     a <- LinStatExpCov(X = ix, y, ...)
+     b <- LECV(Xfactor, y, ...)
```

```
+     d <- LinStatExpCov(X = integer(0), ix = ix, Y = iY2d, iy = iy, ...)
+     return(cmpr(a, b) && cmpr(d, b))
+ }
> stopifnot(
+     testit() && testit(weights = weights) &&
+     testit(subset = subset) && testit(weights = weights, subset = subset) &&
+     testit(block = block) && testit(weights = weights, block = block) &&
+     testit(subset = subset, block = block) &&
+     testit(weights = weights, subset = subset, block = block))
```

All three implementations give the same results.

3.3 Conventions

Functions starting with `R_` are C functions callable via `.Call()` from R. That means they all return `SEXP`. These functions allocate memory handled by R.

Functions starting with `RC_` are C functions with `SEXP` or pointer arguments and possibly an `SEXP` return value.

Functions starting with `C_` only take pointer arguments and return a scalar nor nothing.

Return values (arguments modified by a function) are named `ans`, sometimes with dimension (for example: `PQ_ans`).

3.4 C User Interface

3.4.1 One-Dimensional Case ("1d")

⟨ *User Interface* 31a ⟩ ≡

```
⟨ RC_ExpectationCovarianceStatistic 34 ⟩
⟨ R_ExpectationCovarianceStatistic 32c ⟩
⟨ R_PermutedLinearStatistic 40 ⟩
⟨ R_StandardisePermutedLinearStatistic 42a ⟩
◇
```

Fragment referenced in 24a.

The data are given as $\mathbf{x}_i$ and $\mathbf{y}_i$ for $i = 1, \dots, N$, optionally with weights, subset and blocks. The latter three variables are ignored when specified as `integer(0)`.

⟨ *User Interface Input* 31b ⟩ ≡

```
⟨ R x Input 24d ⟩
⟨ R y Input 25d ⟩
⟨ R weights Input 26c ⟩,
⟨ R subset Input 27b ⟩,
⟨ R block Input 28b ⟩,
◇
```

Fragment referenced in 32b, 34, 38c.

This function can be called from other packages.

"libcoinAPI.h" 32a≡

```
⟨ C Header 166b ⟩
#include <R_ext/Rdynload.h>
#include <libcoin.h>

extern SEXP libcoin_R_ExpectationCovarianceStatistic(
    SEXP x, SEXP y, SEXP weights, SEXP subset, SEXP block, SEXP varonly,
    SEXP tol
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_ExpectationCovarianceStatistic");
    return fun(x, y, weights, subset, block, varonly, tol);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: block 28bd, R_ExpectationCovarianceStatistic 32c, subset 27be, 28a, weights 26c, weights, 26de, x 24d, 25bc, y 25d, 26ab.

⟨ *R_ExpectationCovarianceStatistic Prototype* 32b ⟩ ≡

```
SEXP R_ExpectationCovarianceStatistic
(
    ⟨ User Interface Input 31b ⟩
    SEXP varonly,
    SEXP tol
)
◇
```

Fragment referenced in 23b, 32c.
Uses: R_ExpectationCovarianceStatistic 32c.

The C interface essentially sets-up the necessary memory and calls a C level function for the computations.

⟨ *R_ExpectationCovarianceStatistic* 32c ⟩ ≡

```
⟨ R_ExpectationCovarianceStatistic Prototype 32b ⟩
{
    SEXP ans;

    ⟨ Setup Dimensions 33 ⟩

    PROTECT(ans = RC_init_LECV_1d(P, Q, INTEGER(varonly)[0], B, TYPEOF(x) == INTSXP, REAL(tol)[0]));

    RC_ExpectationCovarianceStatistic(x, y, weights, subset, block, ans);

    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 31a.
Defines: R_ExpectationCovarianceStatistic 6, 32ab, 164, 165.
Uses: B 28c, block 28bd, P 25a, Q 25e, RC_ExpectationCovarianceStatistic 34, 48, RC_init_LECV_1d 160b, subset 27be, 28a, weights 26c, weights, 26de, x 24d, 25bc, y 25d, 26ab.

P, Q and B are first extracted from the data. The case where X is an implicitly specified dummy matrix, the dimension P is the number of levels of x.

⟨ *Setup Dimensions* 33 ⟩ ≡

```
int P, Q, B;

if (TYPEOF(x) == INTSXP) {
    P = NLEVELS(x);
} else {
    P = NCOL(x);
}
Q = NCOL(y);

B = 1;
if (LENGTH(block) > 0)
    B = NLEVELS(block);
◇
```

Fragment referenced in 32c, 40.
Uses: B 28c, block 28bd, NCOL 139c, NLEVELS 140a, P 25a, Q 25e, x 24d, 25bc, y 25d, 26ab.

The core function first computes the linear statistic (as there is no need to pay attention to blocks) and, in a second step, starts a loop over potential blocks.

FIXME: x being an integer (Xfactor) with some 0 elements is not handled correctly (as sumweights doesnt't take this information into account; use subset to exclude these missings (as done in libcoin::LinStatExpCov)

⟨ *RC_ExpectationCovarianceStatistic* 34 ⟩ ≡

```
void RC_ExpectationCovarianceStatistic
(
    ⟨ User Interface Input 31b ⟩
    SEXP ans
) {
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer P Input 25a ⟩;
    ⟨ C integer Q Input 25e ⟩;
    ⟨ C integer B Input 28c ⟩;
    double *sumweights, *table;
    double *ExpInf, *VarInf, *CovInf, *ExpX, *ExpXtotal, *VarX, *CovX;
    double *tmpV, *tmpCV;
    SEXP nullvec, subset_block;

    ⟨ Extract Dimensions 35a ⟩

    ⟨ Compute Linear Statistic 35b ⟩

    ⟨ Setup Memory and Subsets in Blocks 36a ⟩

    /* start with subset[0] */
    R_xlen_t offset = (R_xlen_t) table[0];

    for (int b = 0; b < B; b++) {

        ⟨ Compute Sum of Weights in Block 36b ⟩

        /* don't do anything for empty blocks or blocks with weight 1 */
        if (sumweights[b] > 1) {

            ⟨ Compute Expectation Linear Statistic 37a ⟩

            ⟨ Compute Covariance Influence 37b ⟩

            if (C_get_varonly(ans)) {
                ⟨ Compute Variance Linear Statistic 37c ⟩
            } else {
                ⟨ Compute Covariance Linear Statistic 38a ⟩
            }
        }

        /* next iteration starts with subset[cumsum(table[1:(b + 1)])] */
        offset += (R_xlen_t) table[b + 1];
    }

    ⟨ Compute Variance from Covariance 38b ⟩

    Free(ExpX); Free(VarX); Free(CovX);
    UNPROTECT(2);
}
◇
```

Fragment referenced in 31a.
Defines: `RC_ExpectationCovarianceStatistic` 32c.
Uses: `B` 28c, `C_get_varonly` 152b, `offset` 27d, `subset` 27be, 28a, `sumweights` 27a.

The dimensions are available from the return object:

⟨ *Extract Dimensions* 35a ⟩ ≡

```
P = C_get_P(ans);
Q = C_get_Q(ans);
N = NROW(x);
B = C_get_B(ans);
◇
```

Fragment referenced in 34.
Uses: `B` 28c, `C_get_B` 157a, `C_get_P` 151c, `C_get_Q` 152a, `N` 24bc, `NROW` 139b, `P` 25a, `Q` 25e, `x` 24d, 25bc.

The linear statistic $\mathbf{T}(\mathcal{A})$ can be computed without taking blocks into account.

⟨ *Compute Linear Statistic* 35b ⟩ ≡

```
RC_LinearStatistic(x, N, P, REAL(y), Q, weights, subset,
                   Offset0, XLENGTH(subset),
                   C_get_LinearStatistic(ans));
◇
```

Fragment referenced in 34.
Uses: `C_get_LinearStatistic` 152d, `N` 24bc, `Offset0` 22b, `P` 25a, `Q` 25e, `RC_LinearStatistic` 81d, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc, `y` 25d, 26ab.

We next extract memory from the return object and allocate some additional memory. The most important step is to tabulate blocks and to order the subset with respect to blocks. In absense of block, this just returns subset.

⟨ *Setup Memory and Subsets in Blocks* 36a ⟩ ≡

```
ExpInf = C_get_ExpectationInfluence(ans);
VarInf = C_get_VarianceInfluence(ans);
CovInf = C_get_CovarianceInfluence(ans);
ExpXtotal = C_get_ExpectationX(ans);
for (int p = 0; p < P; p++) ExpXtotal[p] = 0.0;
ExpX = Calloc(P, double);
/* Fix by Joanidis Kristoforos: P > INT_MAX is possible
   for maximally selected statistics (when X is an integer).
   2018-12-13
*/
if (C_get_varonly(ans)) {
    VarX = Calloc(P, double);
    CovX = Calloc(1, double);
} else {
    VarX = Calloc(1, double);
    CovX = Calloc(PP12(P), double);
}
table = C_get_TableBlock(ans);
sumweights = C_get_Sumweights(ans);
PROTECT(nullvec = allocVector(INTSXP, 0));

if (B == 1) {
    table[0] = 0.0;
    table[1] = RC_Sums(N, nullvec, subset, Offset0, XLENGTH(subset));
} else {
    RC_OneTableSums(INTEGER(block), N, B + 1, nullvec, subset, Offset0,
                    XLENGTH(subset), table);
}
if (table[0] > 0)
    error("No missing values allowed in block");
PROTECT(subset_block = RC_order_subset_wrt_block(N, subset, block,
                                                 VECTOR_ELT(ans, TableBlock_SLOT)));
◇
```

Fragment referenced in 34.
Uses: B 28c, block 28bd, C_get_CovarianceInfluence 155a, C_get_ExpectationInfluence 154c, C_get_ExpectationX 154b, C_get_Sumweights 156a, C_get_TableBlock 155c, C_get_VarianceInfluence 155b, C_get_varonly 152b, N 24bc, Offset0 22b, P 25a, PP12 140b, RC_OneTableSums 119a, RC_order_subset_wrt_block 133b, RC_Sums 96a, subset 27be, 28a, sumweights 27a, TableBlock_SLOT 22b.

We compute $\mu(\mathcal{A})$ based on $\mathbb{E}(h \mid S(\mathcal{A}))$ and $\sum_{i \in \mathcal{A}} w_i \mathbf{x}_i$ for the subset given by subset and the bth level of block. The expectation is initialised zero when $b = 0$ and values add-up over blocks.

⟨ *Compute Sum of Weights in Block* 36b ⟩ ≡

```
/* compute sum of weights in block b of subset */
if (table[b + 1] > 0) {
    sumweights[b] = RC_Sums(N, weights, subset_block,
                            offset, (R_xlen_t) table[b + 1]);
} else {
    /* offset = something and Nsubset = 0 means Nsubset = N in
       RC_Sums; catch empty or zero-weight block levels here */
    sumweights[b] = 0.0;
}
◇
```

Fragment referenced in 34.
Uses: `block` 28bd, `N` 24bc, `Nsubset` 27c, `offset` 27d, `RC_Sums` 96a, `subset` 27be, 28a, `sumweights` 27a, `weights` 26c, `weights,` 26de.

⟨ *Compute Expectation Linear Statistic* 37a ⟩ ≡

```
RC_ExpectationInfluence(N, y, Q, weights, subset_block, offset,
                        (R_xlen_t) table[b + 1], sumweights[b], ExpInf + b * Q);
RC_ExpectationX(x, N, P, weights, subset_block, offset,
                (R_xlen_t) table[b + 1], ExpX);
for (int p = 0; p < P; p++) ExpXtotal[p] += ExpX[p];
C_ExpectationLinearStatistic(P, Q, ExpInf + b * Q, ExpX, b,
                             C_get_Expectation(ans));
◇
```

Fragment referenced in 34.
Uses: `C_ExpectationLinearStatistic` 82b, `C_get_Expectation` 153a, `N` 24bc, `offset` 27d, `P` 25a, `Q` 25e, `RC_ExpectationInfluence` 86a, `RC_ExpectationX` 90, `sumweights` 27a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc, `y` 25d, 26ab.

The covariance $\mathbb{V}(h \mid S(\mathcal{A}))$ is now computed for the subset given by subset and the bth level of block. Note that `CovInf` stores the values for each block in the return object (for later reuse).

⟨ *Compute Covariance Influence* 37b ⟩ ≡

```
/* C_ordered_Xfactor and C_unordered_Xfactor need both VarInf and CovInf */
RC_CovarianceInfluence(N, y, Q, weights, subset_block, offset,
                       (R_xlen_t) table[b + 1], ExpInf + b * Q, sumweights[b],
                       !DoVarOnly, CovInf + b * Q * (Q + 1) / 2);
/* extract variance from covariance */
tmpCV = CovInf + b * Q * (Q + 1) / 2;
tmpV = VarInf + b * Q;
for (int q = 0; q < Q; q++) tmpV[q] = tmpCV[S(q, q, Q)];
◇
```

Fragment referenced in 34.
Uses: `C_ordered_Xfactor` 73, `C_unordered_Xfactor` 78, `DoVarOnly` 22b, `N` 24bc, `offset` 27d, `Q` 25e, `RC_CovarianceInfluence` 88a, `S` 22a, `sumweights` 27a, `weights` 26c, `weights,` 26de, `y` 25d, 26ab.

We can now compute the variance or covariance of the linear statistic $\Sigma(\mathcal{A})$:

⟨ *Compute Variance Linear Statistic* 37c ⟩ ≡

```
RC_CovarianceX(x, N, P, weights, subset_block, offset,
               (R_xlen_t) table[b + 1], ExpX, DoVarOnly, VarX);
C_VarianceLinearStatistic(P, Q, VarInf + b * Q, ExpX, VarX, sumweights[b],
                          b, C_get_Variance(ans));
◇
```

Fragment referenced in 34.
Uses: C_get_Variance 153b, C_VarianceLinearStatistic 84, DoVarOnly 22b, N 24bc, offset 27d, P 25a, Q 25e, RC_CovarianceX 93a, sumweights 27a, weights 26c, weights, 26de, x 24d, 25bc.

⟨ *Compute Covariance Linear Statistic* 38a ⟩ ≡

```
RC_CovarianceX(x, N, P, weights, subset_block, offset,
               (R_xlen_t) table[b + 1], ExpX, !DoVarOnly, CovX);
C_CovarianceLinearStatistic(P, Q, CovInf + b * Q * (Q + 1) / 2,
                            ExpX, CovX, sumweights[b], b,
                            C_get_Covariance(ans));
◇
```

Fragment referenced in 34.
Uses: C_CovarianceLinearStatistic 83, C_get_Covariance 154a, DoVarOnly 22b, N 24bc, offset 27d, P 25a, Q 25e, RC_CovarianceX 93a, sumweights 27a, weights 26c, weights, 26de, x 24d, 25bc.

⟨ *Compute Variance from Covariance* 38b ⟩ ≡

```
/* always return variances */
if (!C_get_varonly(ans)) {
    for (int p = 0; p < mPQB(P, Q, 1); p++)
        C_get_Variance(ans)[p] = C_get_Covariance(ans)[S(p, p, mPQB(P, Q, 1))];
}
◇
```

Fragment referenced in 34.
Uses: C_get_Covariance 154a, C_get_Variance 153b, C_get_varonly 152b, mPQB 141a, P 25a, Q 25e, S 22a.

The computation of permuted linear statistics is done outside this general function. The user interface is the same, except for an additional number of permutations to be specified.

⟨ *R_PermutedLinearStatistic Prototype* 38c ⟩ ≡

```
SEXP R_PermutedLinearStatistic
(
    ⟨ User Interface Input 31b ⟩
    SEXP nresample
)
◇
```

Fragment referenced in 23b, 40.
Uses: R_PermutedLinearStatistic 40.

"libcoinAPI.h" 38d≡

```
extern SEXP libcoin_R_PermutedLinearStatistic(
    SEXP x, SEXP y, SEXP weights, SEXP subset, SEXP block, SEXP nresample
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_PermutedLinearStatistic");
    return fun(x, y, weights, subset, block, nresample);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: `block` 28bd, `R_PermutedLinearStatistic` 40, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc, `y` 25d, 26ab.

The dimensions are extracted from the data in the same ways as above. The function differentiates between the absense and presense of blocks. Weights are removed by expanding subset accordingly. Once within-block permutations were set-up the Kronecker product of `X` and `Y` is computed. Note that this function returns the matrix of permuted linear statistics; the R interface assigns this matrix to the corresponding element of the `LinStatExpCov` object (because we are not allowed to modify existing R objects at C level).

⟨ *R_PermutedLinearStatistic* 40 ⟩ ≡

```
    ⟨ R_PermutedLinearStatistic Prototype 38c ⟩
    {
        SEXP ans, expand_subset, block_subset, perm, tmp, blockTable;
        double *linstat;
        int PQ;
        ⟨ C integer N Input 24c ⟩;
        ⟨ C integer Nsubset Input 27c ⟩;
        R_xlen_t inresample;

        ⟨ Setup Dimensions 33 ⟩
        PQ = mPQB(P, Q, 1);
        N = NROW(y);
        inresample = (R_xlen_t) REAL(nresample)[0];

        PROTECT(ans = allocMatrix(REALSXP, PQ, inresample));
        PROTECT(expand_subset = RC_setup_subset(N, weights, subset));
        Nsubset = XLENGTH(expand_subset);
        PROTECT(tmp = allocVector(REALSXP, Nsubset));
        PROTECT(perm = allocVector(REALSXP, Nsubset));

        GetRNGstate();
        if (B == 1) {
            for (R_xlen_t np = 0; np < inresample; np++) {
                ⟨ Setup Linear Statistic 41a ⟩
                C_doPermute(REAL(expand_subset), Nsubset, REAL(tmp), REAL(perm));
                RC_KronSums_Permutation(x, NROW(x), P, REAL(y), Q, expand_subset,
                                        Offset0, Nsubset, perm, linstat);
            }
        } else {
            PROTECT(blockTable = allocVector(REALSXP, B + 1));
            /* same as RC_OneTableSums(block, noweights, expand_subset) */
            RC_OneTableSums(INTEGER(block), XLENGTH(block), B + 1, weights, subset, Offset0,
                            XLENGTH(subset), REAL(blockTable));
            PROTECT(block_subset = RC_order_subset_wrt_block(XLENGTH(block), expand_subset,
                                                             block, blockTable));

            for (R_xlen_t np = 0; np < inresample; np++) {
                ⟨ Setup Linear Statistic 41a ⟩
                C_doPermuteBlock(REAL(block_subset), Nsubset, REAL(blockTable),
                                 B + 1, REAL(tmp), REAL(perm));
                RC_KronSums_Permutation(x, NROW(x), P, REAL(y), Q, block_subset,
                                        Offset0, Nsubset, perm, linstat);
            }
            UNPROTECT(2);
        }
        PutRNGstate();

        UNPROTECT(4);
        return(ans);
    }
    ◇
```

Fragment referenced in 31a.
Defines: `R_PermutedLinearStatistic` 6, 38cd, 164, 165.
Uses: `B` 28c, `block` 28bd, `blockTable` 28e, `C_doPermute` 137b, `C_doPermuteBlock` 138b, `mPQB` 141a, `N` 24bc, `NROW` 139b, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `Q` 25e, `RC_KronSums_Permutation` 110a, `RC_OneTableSums` 119a, `RC_order_subset_wrt_block` 133b, `RC_setup_subset` 136a, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *Setup Linear Statistic* 41a ⟩ ≡

```
if (np % 256 == 0) R_CheckUserInterrupt();
linstat = REAL(ans) + PQ * np;
for (int p = 0; p < PQ; p++)
    linstat[p] = 0.0;
◇
```

Fragment referenced in 40, 51.

"libcoinAPI.h" 41b≡

```
extern SEXP libcoin_R_StandardisePermutedLinearStatistic(
    SEXP LECV
) {
    static SEXP(*fun)(SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP))
            R_GetCCallable("libcoin", "R_StandardisePermutedLinearStatistic");
    return fun(LECV);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: LECV 151b.

This small function takes an object containing permuted linear statistics and returns the matrix of standardised linear statistics.

⟨ *R_StandardisePermutedLinearStatistic Prototype* 41c ⟩ ≡

```
SEXP R_StandardisePermutedLinearStatistic
(
    SEXP LECV
)
◇
```

Fragment referenced in 23b, 42a.
Uses: LECV 151b.

⟨ *R_StandardisePermutedLinearStatistic* 42a ⟩ ≡

```
⟨ R_StandardisePermutedLinearStatistic Prototype 41c ⟩
{
    SEXP ans;
    R_xlen_t nresample = C_get_nresample(LECV);
    double *ls;
    if (!nresample) return(R_NilValue);
    int PQ = C_get_P(LECV) * C_get_Q(LECV);

    PROTECT(ans = allocMatrix(REALSXP, PQ, nresample));

    for (R_xlen_t np = 0; np < nresample; np++) {
        ls = REAL(ans) + PQ * np;
        /* copy first; standarisation is in place */
        for (int p = 0; p < PQ; p++)
            ls[p] = C_get_PermutedLinearStatistic(LECV)[p + PQ * np];
        if (C_get_varonly(LECV)) {
            C_standardise(PQ, ls, C_get_Expectation(LECV),
                          C_get_Variance(LECV), 1, C_get_tol(LECV));
        } else {
            C_standardise(PQ, ls, C_get_Expectation(LECV),
                          C_get_Covariance(LECV), 0, C_get_tol(LECV));
        }
    }
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 31a.
Uses: `C_get_Covariance` 154a, `C_get_Expectation` 153a, `C_get_nresample` 157b, `C_get_P` 151c, `C_get_PermutedLinearStatistic` 157c, `C_get_Q` 152a, `C_get_tol` 157d, `C_get_Variance` 153b, `C_get_varonly` 152b, `C_standardise` 67a, `LECV` 151b.

3.4.2 Two-Dimensional Case ("2d")

⟨ *2d User Interface* 42b ⟩ ≡

```
⟨ RC_ExpectationCovarianceStatistic_2d 48 ⟩
⟨ R_ExpectationCovarianceStatistic_2d 44 ⟩
⟨ R_PermutedLinearStatistic_2d 51 ⟩
◇
```

Fragment referenced in 24a.

⟨ *2d User Interface Input* 42c ⟩ ≡

```
⟨ R x Input 24d ⟩
SEXP ix,
⟨ R y Input 25d ⟩
SEXP iy,
⟨ R weights Input 26c ⟩,
⟨ R subset Input 27b ⟩,
⟨ R block Input 28b ⟩,
◇
```

Fragment referenced in 43a, 48.

⟨ *R_ExpectationCovarianceStatistic_2d Prototype* 43a ⟩ ≡

```
SEXP R_ExpectationCovarianceStatistic_2d
(
    ⟨ 2d User Interface Input 42c ⟩
    SEXP varonly,
    SEXP tol
)
◇
```

Fragment referenced in 23b, 44.
Uses: `R_ExpectationCovarianceStatistic_2d` 44.

"libcoinAPI.h" 43b≡

```
extern SEXP libcoin_R_ExpectationCovarianceStatistic_2d(
    SEXP x, SEXP ix, SEXP y, SEXP iy, SEXP weights, SEXP subset, SEXP block,
    SEXP varonly, SEXP tol
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_ExpectationCovarianceStatistic_2d");
    return fun(x, ix, y, iy, weights, subset, block, varonly, tol);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: `block` 28bd, `R_ExpectationCovarianceStatistic_2d` 44, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *R_ExpectationCovarianceStatistic_2d* 44 ⟩ ≡

```
⟨ R_ExpectationCovarianceStatistic_2d Prototype 43a ⟩
{
    SEXP ans;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;
    int Xfactor;

    N = XLENGTH(ix);
    Nsubset = XLENGTH(subset);
    Xfactor = XLENGTH(x) == 0;

    ⟨ Setup Dimensions 2d 45a ⟩

    PROTECT(ans = RC_init_LECV_2d(P, Q, INTEGER(varonly)[0],
                                  Lx, Ly, B, Xfactor, REAL(tol)[0]));

    if (B == 1) {
        RC_TwoTableSums(INTEGER(ix), N, Lx + 1, INTEGER(iy), Ly + 1,
                        weights, subset, Offset0, Nsubset,
                        C_get_Table(ans));
    } else {
        RC_ThreeTableSums(INTEGER(ix), N, Lx + 1, INTEGER(iy), Ly + 1,
                          INTEGER(block), B, weights, subset, Offset0, Nsubset,
                          C_get_Table(ans));
    }
    RC_ExpectationCovarianceStatistic_2d(x, ix, y, iy, weights,
                                         subset, block, ans);

    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 42b.
Defines: `R_ExpectationCovarianceStatistic_2d` 8, 43ab, 164, 165.
Uses: `B` 28c, `block` 28bd, `C_get_Table` 156b, `N` 24bc, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `Q` 25e, `RC_init_LECV_2d` 161, `RC_ThreeTableSums` 128b, `RC_TwoTableSums` 123b, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *Setup Dimensions 2d* 45a ⟩ ≡

```
int P, Q, B, Lx, Ly;

if (XLENGTH(x) == 0) {
    P = NLEVELS(ix);
} else {
    P = NCOL(x);
}
Q = NCOL(y);

B = 1;
if (XLENGTH(block) > 0)
    B = NLEVELS(block);

Lx = NLEVELS(ix);
Ly = NLEVELS(iy);
◇
```

Fragment referenced in 44, 51.
Uses: `B` 28c, `block` 28bd, `NCOL` 139c, `NLEVELS` 140a, `P` 25a, `Q` 25e, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *Linear Statistic 2d* 45b ⟩ ≡

```
if (Xfactor) {
    for (int j = 1; j < Lyp1; j++) { /* j = 0 means NA */
        for (int i = 1; i < Lxp1; i++) { /* i = 0 means NA */
            for (int q = 0; q < Q; q++)
                linstat[q * (Lxp1 - 1) + (i - 1)] +=
                    btab[j * Lxp1 + i] * REAL(y)[q * Lyp1 + j];
        }
    }
} else {
    for (int p = 0; p < P; p++) {
        for (int q = 0; q < Q; q++) {
            int qPp = q * P + p;
            int qLy = q * Lyp1;
            for (int i = 0; i < Lxp1; i++) {
                int pLxi = p * Lxp1 + i;
                for (int j = 0; j < Lyp1; j++)
                    linstat[qPp] += REAL(y)[qLy + j] * REAL(x)[pLxi] * btab[j * Lxp1 + i];
            }
        }
    }
}
◇
```

Fragment referenced in 48, 53a.
Uses: `P` 25a, `Q` 25e, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *2d Total Table* 46a ⟩ ≡

```
for (int i = 0; i < Lxp1 * Lyp1; i++)
    table2d[i] = 0.0;
for (int b = 0; b < B; b++) {
    for (int i = 0; i < Lxp1; i++) {
        for (int j = 0; j < Lyp1; j++)
            table2d[j * Lxp1 + i] += table[b * Lxp1 * Lyp1 + j * Lxp1 + i];
    }
}
◇
```

Fragment referenced in 48.
Uses: `B` 28c.

⟨ *Col Row Total Sums* 46b ⟩ ≡

```
/* Remember: first row / column count NAs */
/* column sums */
for (int q = 1; q < Lyp1; q++) {
    csum[q] = 0;
    for (int p = 1; p < Lxp1; p++)
        csum[q] += btab[q * Lxp1 + p];
}
csum[0] = 0; /* NA */
/* row sums */
for (int p = 1; p < Lxp1; p++) {
    rsum[p] = 0;
    for (int q = 1; q < Lyp1; q++)
        rsum[p] += btab[q * Lxp1 + p];
}
rsum[0] = 0; /* NA */
/* total sum */
sumweights[b] = 0;
for (int i = 1; i < Lxp1; i++) sumweights[b] += rsum[i];
◇
```

Fragment referenced in 48, 51.
Uses: `sumweights` 27a.

⟨ *2d Expectation* 46c ⟩ ≡

```
RC_ExpectationInfluence(NROW(y), y, Q, Rcsum, subset, Offset0, 0, sumweights[b], ExpInf);

if (LENGTH(x) == 0) {
    for (int p = 0; p < P; p++)
        ExpX[p] = rsum[p + 1];
    } else {
        RC_ExpectationX(x, NROW(x), P, Rrsum, subset, Offset0, 0, ExpX);
}

C_ExpectationLinearStatistic(P, Q, ExpInf, ExpX, b, C_get_Expectation(ans));
◇
```

Fragment referenced in 48.
Uses: `C_ExpectationLinearStatistic` 82b, `C_get_Expectation` 153a, `NROW` 139b, `Offset0` 22b, `P` 25a, `Q` 25e, `RC_ExpectationInfluence` 86a, `RC_ExpectationX` 90, `subset` 27be, 28a, `sumweights` 27a, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *2d Covariance* 47 ⟩ ≡

```
/* C_ordered_Xfactor needs both VarInf and CovInf */
RC_CovarianceInfluence(NROW(y), y, Q, Rcsum, subset, Offset0, 0, ExpInf, sumweights[b],
                       !DoVarOnly, C_get_CovarianceInfluence(ans));
for (int q = 0; q < Q; q++)
    C_get_VarianceInfluence(ans)[q] = C_get_CovarianceInfluence(ans)[S(q, q, Q)];

if (C_get_varonly(ans)) {
    if (LENGTH(x) == 0) {
        for (int p = 0; p < P; p++) CovX[p] = ExpX[p];
    } else {
        RC_CovarianceX(x, NROW(x), P, Rrsum, subset, Offset0, 0, ExpX, DoVarOnly, CovX);
    }
    C_VarianceLinearStatistic(P, Q, C_get_VarianceInfluence(ans),
                              ExpX, CovX, sumweights[b], b,
                              C_get_Variance(ans));
} else {
    if (LENGTH(x) == 0) {
        for (int p = 0; p < PP12(P); p++) CovX[p] = 0.0;
        for (int p = 0; p < P; p++) CovX[S(p, p, P)] = ExpX[p];
    } else {
        RC_CovarianceX(x, NROW(x), P, Rrsum, subset, Offset0, 0, ExpX, !DoVarOnly, CovX);
    }
    C_CovarianceLinearStatistic(P, Q, C_get_CovarianceInfluence(ans),
                                ExpX, CovX, sumweights[b], b,
                                C_get_Covariance(ans));
}
◇
```

Fragment referenced in 48.

Uses: C_CovarianceLinearStatistic 83, C_get_Covariance 154a, C_get_CovarianceInfluence 155a, C_get_Variance 153b, C_get_VarianceInfluence 155b, C_get_varonly 152b, C_ordered_Xfactor 73, C_VarianceLinearStatistic 84, DoVarOnly 22b, NROW 139b, Offset0 22b, P 25a, PP12 140b, Q 25e, RC_CovarianceInfluence 88a, RC_CovarianceX 93a, S 22a, subset 27be, 28a, sumweights 27a, x 24d, 25bc, y 25d, 26ab.

⟨ *RC_ExpectationCovarianceStatistic_2d* 48 ⟩ ≡

```
void RC_ExpectationCovarianceStatistic_2d
(
    ⟨ 2d User Interface Input 42c ⟩
    SEXP ans
) {
    ⟨ 2d Memory 49 ⟩

    ⟨ 2d Total Table 46a ⟩

    linstat = C_get_LinearStatistic(ans);
    for (int p = 0; p < mPQB(P, Q, 1); p++)
        linstat[p] = 0.0;

    for (int b = 0; b < B; b++) {
        btab = table + Lxp1 * Lyp1 * b;

        ⟨ Linear Statistic 2d 45b ⟩

        ⟨ Col Row Total Sums 46b ⟩

        ⟨ 2d Expectation 46c ⟩

        ⟨ 2d Covariance 47 ⟩
    }

    /* always return variances */
    if (!C_get_varonly(ans)) {
        for (int p = 0; p < mPQB(P, Q, 1); p++)
            C_get_Variance(ans)[p] = C_get_Covariance(ans)[S(p, p, mPQB(P, Q, 1))];
    }

    Free(CovX);
    Free(table2d);
    UNPROTECT(2);
}
◇
```

Fragment referenced in 42b.
Defines: `RC_ExpectationCovarianceStatistic` 32c, 34.
Uses: `B` 28c, `C_get_Covariance` 154a, `C_get_LinearStatistic` 152d, `C_get_Variance` 153b, `C_get_varonly` 152b, `mPQB` 141a, `P` 25a, `Q` 25e, `S` 22a.

⟨ *2d Memory* 49 ⟩ ≡

```
SEXP Rcsum, Rrsum;
int P, Q, Lxp1, Lyp1, B, Xfactor;
double *ExpInf, *ExpX, *CovX;
double *table, *table2d, *csum, *rsum, *sumweights, *btab, *linstat;

P = C_get_P(ans);
Q = C_get_Q(ans);

ExpInf = C_get_ExpectationInfluence(ans);
ExpX = C_get_ExpectationX(ans);
table = C_get_Table(ans);
sumweights = C_get_Sumweights(ans);

Lxp1 = C_get_dimTable(ans)[0];
Lyp1 = C_get_dimTable(ans)[1];
B = C_get_B(ans);
Xfactor = C_get_Xfactor(ans);

if (C_get_varonly(ans)) {
    CovX = Calloc(P, double);
} else {
    CovX = Calloc(PP12(P), double);
}

table2d = Calloc(Lxp1 * Lyp1, double);
PROTECT(Rcsum = allocVector(REALSXP, Lyp1));
csum = REAL(Rcsum);
PROTECT(Rrsum = allocVector(REALSXP, Lxp1));
rsum = REAL(Rrsum);
◇
```

Fragment referenced in 48.
Uses: B 28c, C_get_B 157a, C_get_dimTable 156c, C_get_ExpectationInfluence 154c, C_get_ExpectationX 154b, C_get_P 151c, C_get_Q 152a, C_get_Sumweights 156a, C_get_Table 156b, C_get_varonly 152b, C_get_Xfactor 152c, P 25a, PP12 140b, Q 25e, sumweights 27a.

```
> LinStatExpCov(X = iX2d, ix = ix, Y = iY2d, iy = iy,
+                weights = weights, subset = subset, nresample = 10)$PermutedLinearStatistic
           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]      [,7]
 [1,] 15.486226 15.432786 15.474636 15.434733 15.515989 15.421226 15.523577
 [2,]  7.864472  7.948006  8.105228  8.390763  8.212044  8.493673  8.415919
 [3,] 12.398382 12.290212 11.905712 12.506639 12.143855 12.549147 12.590900
 [4,] 31.244688 31.476627 31.257920 31.264541 31.096744 31.405390 31.259421
 [5,] 17.500047 17.688850 17.055915 15.065147 16.586396 15.315949 16.382058
 [6,] 24.466142 24.647923 25.464893 24.239801 25.488434 24.296588 23.694321
 [7,] 43.423057 43.421097 43.330443 43.612924 43.424099 43.430492 43.309780
 [8,] 24.311651 23.474319 22.844531 23.490053 23.541204 22.749540 22.388328
 [9,] 34.180046 34.430423 35.397534 33.988759 34.366957 33.293748 33.389741
[10,] 13.461330 13.490553 13.492064 13.434007 13.447127 13.491634 13.476994
[11,]  6.973432  7.169633  7.259611  6.943487  7.017767  7.094398  7.241183
[12,] 10.672723 10.658055 10.574382 10.675107 10.743783 10.837748 10.788257
           [,8]      [,9]     [,10]
 [1,] 15.434192 15.491818 15.398248
 [2,]  7.834800  8.223809  7.925817
```

```
 [3,] 12.362877 11.997518 12.169918
 [4,] 31.510352 31.284964 31.576643
 [5,] 18.211173 16.969768 17.197270
 [6,] 23.773081 25.183959 24.742788
 [7,] 43.375471 43.374905 43.496870
 [8,] 23.445718 22.372659 23.729797
 [9,] 34.264857 35.341197 34.487409
[10,] 13.498456 13.473376 13.482370
[11,]  7.221054  7.329256  7.097392
[12,] 10.669965 10.540119 10.702889
```

⟨ *R_PermutedLinearStatistic_2d Prototype* 50a ⟩ ≡

```
SEXP R_PermutedLinearStatistic_2d
(
    ⟨ R x Input 24d ⟩
    SEXP ix,
    ⟨ R y Input 25d ⟩
    SEXP iy,
    ⟨ R block Input 28b ⟩,
    SEXP nresample,
    SEXP itable
)
◇
```

Fragment referenced in 23b, 51.
Uses: `R_PermutedLinearStatistic_2d` 51.

"libcoinAPI.h" 50b≡

```
extern SEXP libcoin_R_PermutedLinearStatistic_2d(
    SEXP x, SEXP ix, SEXP y, SEXP iy, SEXP block, SEXP nresample,
    SEXP itable
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_PermutedLinearStatistic_2d");
    return fun(x, ix, y, iy, block, nresample, itable);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: `block` 28bd, `R_PermutedLinearStatistic_2d` 51, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *R_PermutedLinearStatistic_2d* 51 ⟩ ≡

```
    ⟨ R_PermutedLinearStatistic_2d Prototype 50a ⟩
    {
        SEXP ans, Ritable;
        int *csum, *rsum, *sumweights, *jwork, *table, *rtable2, maxn = 0, Lxp1, Lyp1, *btab, PQ, Xfactor;
        R_xlen_t inresample;
        double *fact, *linstat;

        ⟨ Setup Dimensions 2d 45a ⟩

        PQ = mPQB(P, Q, 1);
        Xfactor = XLENGTH(x) == 0;
        Lxp1 = Lx + 1;
        Lyp1 = Ly + 1;
        inresample = (R_xlen_t) REAL(nresample)[0];

        PROTECT(ans = allocMatrix(REALSXP, PQ, inresample));

        ⟨ Setup Working Memory 52b ⟩

        ⟨ Convert Table to Integer 52a ⟩

        for (int b = 0; b < B; b++) {
            btab = INTEGER(Ritable) + Lxp1 * Lyp1 * b;
            ⟨ Col Row Total Sums 46b ⟩
            if (sumweights[b] > maxn) maxn = sumweights[b];
        }

        ⟨ Setup Log-Factorials 52c ⟩

        GetRNGstate();

        for (R_xlen_t np = 0; np < inresample; np++) {
            ⟨ Setup Linear Statistic 41a ⟩

            for (int p = 0; p < Lxp1 * Lyp1; p++)
                table[p] = 0;

            for (int b = 0; b < B; b++) {
                ⟨ Compute Permuted Linear Statistic 2d 53a ⟩
            }
        }

        PutRNGstate();

        Free(csum); Free(rsum); Free(sumweights); Free(rtable2);
        Free(jwork); Free(fact); Free(table);
        UNPROTECT(2);
        return(ans);
    }
    ◇
```

Fragment referenced in 42b.
Defines: `R_PermutedLinearStatistic_2d` 8, 50ab, 52a, 164, 165.
Uses: `B` 28c, `mPQB` 141a, `P` 25a, `Q` 25e, `sumweights` 27a, `x` 24d, 25bc.

⟨ *Convert Table to Integer* 52a ⟩ ≡

```
PROTECT(Ritable = allocVector(INTSXP, LENGTH(itable)));
for (int i = 0; i < LENGTH(itable); i++) {
    if (REAL(itable)[i] > INT_MAX)
        error("cannot deal with weights larger INT_MAX in R_PermutedLinearStatistic_2d");
    INTEGER(Ritable)[i] = (int) REAL(itable)[i];
}
◇
```

Fragment referenced in 51.
Uses: `R_PermutedLinearStatistic_2d` 51, `weights` 26c.

⟨ *Setup Working Memory* 52b ⟩ ≡

```
csum = Calloc(Lyp1 * B, int);
rsum = Calloc(Lxp1 * B, int);
sumweights = Calloc(B, int);
table = Calloc(Lxp1 * Lyp1, int);
rtable2 = Calloc(Lx * Ly , int);
jwork = Calloc(Lyp1, int);
◇
```

Fragment referenced in 51.
Uses: `B` 28c, `sumweights` 27a.

⟨ *Setup Log-Factorials* 52c ⟩ ≡

```
fact = Calloc(maxn + 1, double);
/* Calculate log-factorials.  fact[i] = lgamma(i+1) */
fact[0] = fact[1] = 0.;
for(int j = 2; j <= maxn; j++)
    fact[j] = fact[j - 1] + log(j);
◇
```

Fragment referenced in 51.

Note: the interface to `S_rcont2` changed in R 4.1-0.

⟨ *Compute Permuted Linear Statistic 2d* 53a ⟩ ≡

```
#if defined(R_VERSION) && R_VERSION >= R_Version(4, 1, 0)
            S_rcont2(Lx, Ly,
                    rsum + Lxp1 * b + 1,
                    csum + Lyp1 * b + 1,
                    sumweights[b], fact, jwork, rtable2);
#else
            S_rcont2(&Lx, &Ly, rsum + Lxp1 * b + 1,
                     csum + Lyp1 *b + 1, sumweights + b, fact, jwork, rtable2);
#endif

for (int j1 = 1; j1 <= Lx; j1++) {
    for (int j2 = 1; j2 <= Ly; j2++)
        table[j2 * Lxp1 + j1] = rtable2[(j2 - 1) * Lx + (j1 - 1)];
}
btab = table;
```
⟨ *Linear Statistic 2d* 45b ⟩
◇

Fragment referenced in 51.
Uses: `sumweights` 27a.

3.5 Tests

⟨ *Tests* 53b ⟩ ≡

⟨ *R_QuadraticTest* 55 ⟩
⟨ *R_MaximumTest* 57 ⟩
⟨ *R_MaximallySelectedTest* 59 ⟩
◇

Fragment referenced in 24a.

"libcoinAPI.h" 54a≡

```
extern SEXP libcoin_R_QuadraticTest(
    SEXP LEV, SEXP pvalue, SEXP lower, SEXP give_log, SEXP PermutedStatistics
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_QuadraticTest");
    return fun(LEV, pvalue, lower, give_log, PermutedStatistics);
}

extern SEXP libcoin_R_MaximumTest(
    SEXP LEV, SEXP alternative, SEXP pvalue, SEXP lower, SEXP give_log,
    SEXP PermutedStatistics, SEXP maxpts, SEXP releps, SEXP abseps
) {
  static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_MaximumTest");
    return fun(LEV, alternative, pvalue, lower, give_log, PermutedStatistics, maxpts, releps,
               abseps);
}

extern SEXP libcoin_R_MaximallySelectedTest(
    SEXP LEV, SEXP ordered, SEXP teststat, SEXP minbucket, SEXP lower, SEXP give_log
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP, SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_MaximallySelectedTest");
    return fun(LEV, ordered, teststat, minbucket, lower, give_log);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.

⟨ *R_QuadraticTest Prototype* 54b ⟩ ≡

```
SEXP R_QuadraticTest
(
    ⟨ R LECV Input 151b ⟩,
    SEXP pvalue,
    SEXP lower,
    SEXP give_log,
    SEXP PermutedStatistics
)
◇
```

Fragment referenced in 23b, 55.

⟨ *R_QuadraticTest* 55 ⟩ ≡

```
⟨ R_QuadraticTest Prototype 54b ⟩
{
    SEXP ans, stat, pval, names, permstat;
    double *MPinv, *ls, st, pst, *ex;
    int rank, P, Q, PQ, greater = 0;
    R_xlen_t nresample;

    ⟨ Setup Test Memory 56a ⟩

    MPinv = Calloc(PP12(PQ), double); /* was: C_get_MPinv(LECV); */
    C_MPinv_sym(C_get_Covariance(LECV), PQ, C_get_tol(LECV), MPinv, &rank);

    REAL(stat)[0] = C_quadform(PQ, C_get_LinearStatistic(LECV),
                               C_get_Expectation(LECV), MPinv);

    if (!PVALUE) {
        UNPROTECT(2);
        Free(MPinv);
        return(ans);
    }

    if (C_get_nresample(LECV) == 0) {
        REAL(pval)[0] = C_chisq_pvalue(REAL(stat)[0], rank, LOWER, GIVELOG);
    } else {
        nresample = C_get_nresample(LECV);
        ls = C_get_PermutedLinearStatistic(LECV);
        st = REAL(stat)[0];
        ex = C_get_Expectation(LECV);
        greater = 0;
        for (R_xlen_t np = 0; np < nresample; np++) {
            pst = C_quadform(PQ, ls + PQ * np, ex, MPinv);
            if (GE(pst, st, C_get_tol(LECV)))
                greater++;
            if (PSTAT) REAL(permstat)[np] = pst;
        }
        REAL(pval)[0] = C_perm_pvalue(greater, nresample, LOWER, GIVELOG);
    }

    UNPROTECT(2);
    Free(MPinv);
    return(ans);
}
◇
```

Fragment referenced in 53b.
Uses: `C_chisq_pvalue` 67c, `C_get_Covariance` 154a, `C_get_Expectation` 153a, `C_get_LinearStatistic` 152d, `C_get_nresample` 157b, `C_get_PermutedLinearStatistic` 157c, `C_get_tol` 157d, `C_perm_pvalue` 68, `C_quadform` 65, `GE` 22a, `LECV` 151b, `P` 25a, `PP12` 140b, `Q` 25e.

⟨ *Setup Test Memory* 56a ⟩ ≡

```
P = C_get_P(LECV);
Q = C_get_Q(LECV);
PQ = mPQB(P, Q, 1);

if (C_get_varonly(LECV) && PQ > 1)
        error("cannot compute adjusted p-value based on variances only");
/*  if (C_get_nresample(LECV) > 0 && INTEGER(PermutedStatistics)[0]) { */
    PROTECT(ans = allocVector(VECSXP, 3));
    PROTECT(names = allocVector(STRSXP, 3));
    SET_VECTOR_ELT(ans, 2, permstat = allocVector(REALSXP, C_get_nresample(LECV)));
    SET_STRING_ELT(names, 2, mkChar("PermutedStatistics"));
/*  } else {
    PROTECT(ans = allocVector(VECSXP, 2));
    PROTECT(names = allocVector(STRSXP, 2));
}
*/
SET_VECTOR_ELT(ans, 0, stat = allocVector(REALSXP, 1));
SET_STRING_ELT(names, 0, mkChar("TestStatistic"));
SET_VECTOR_ELT(ans, 1, pval = allocVector(REALSXP, 1));
SET_STRING_ELT(names, 1, mkChar("p.value"));
namesgets(ans, names);
REAL(pval)[0] = NA_REAL;
int LOWER = INTEGER(lower)[0];
int GIVELOG = INTEGER(give_log)[0];
int PVALUE = INTEGER(pvalue)[0];
int PSTAT = INTEGER(PermutedStatistics)[0];
◇
```

Fragment referenced in 55, 57.
Uses: `C_get_nresample` 157b, `C_get_P` 151c, `C_get_Q` 152a, `C_get_varonly` 152b, `LECV` 151b, `mPQB` 141a, `P` 25a, `Q` 25e.

⟨ *R_MaximumTest Prototype* 56b ⟩ ≡

```
SEXP R_MaximumTest
(
    ⟨ R LECV Input 151b ⟩,
    SEXP alternative,
    SEXP pvalue,
    SEXP lower,
    SEXP give_log,
    SEXP PermutedStatistics,
    SEXP maxpts,
    SEXP releps,
    SEXP abseps
)
◇
```

Fragment referenced in 23b, 57.

⟨ *R_MaximumTest* 57 ⟩ ≡

```
⟨ R_MaximumTest Prototype 56b ⟩
{
    SEXP ans, stat, pval, names, permstat;
    double st, pst, *ex, *cv, *ls, tl;
    int P, Q, PQ, vo, alt, greater;
    R_xlen_t nresample;

    ⟨ Setup Test Memory 56a ⟩

    if (C_get_varonly(LECV)) {
        cv = C_get_Variance(LECV);
    } else {
        cv = C_get_Covariance(LECV);
    }
    REAL(stat)[0] = C_maxtype(PQ, C_get_LinearStatistic(LECV),
        C_get_Expectation(LECV), cv, C_get_varonly(LECV), C_get_tol(LECV),
        INTEGER(alternative)[0]);
    if (!PVALUE) {
        UNPROTECT(2);
        return(ans);
    }

    if (C_get_nresample(LECV) == 0) {
        if (C_get_varonly(LECV) && PQ > 1) {
            REAL(pval)[0] = NA_REAL;
            UNPROTECT(2);
            return(ans);
        }
        REAL(pval)[0] = C_maxtype_pvalue(REAL(stat)[0], cv,
            PQ, INTEGER(alternative)[0], LOWER, GIVELOG, INTEGER(maxpts)[0],
            REAL(releps)[0], REAL(abseps)[0], C_get_tol(LECV));
    } else {
        nresample = C_get_nresample(LECV);
        ls = C_get_PermutedLinearStatistic(LECV);
        ex = C_get_Expectation(LECV);
        vo = C_get_varonly(LECV);
        alt = INTEGER(alternative)[0];
        st = REAL(stat)[0];
        tl = C_get_tol(LECV);
        greater = 0;
        for (R_xlen_t np = 0; np < nresample; np++) {
            pst = C_maxtype(PQ, ls + PQ * np, ex, cv, vo, tl, alt);
            if (alt == ALTERNATIVE_less) {
                if (LE(pst, st, tl)) greater++;
            } else {
                if (GE(pst, st, tl)) greater++;
            }
            if (PSTAT) REAL(permstat)[np] = pst;
        }
        REAL(pval)[0] = C_perm_pvalue(greater, nresample, LOWER, GIVELOG);
    }
    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 53b.

Uses: `C_get_Covariance` 154a, `C_get_Expectation` 153a, `C_get_LinearStatistic` 152d, `C_get_nresample` 157b, `C_get_PermutedLinearStatistic` 157c, `C_get_tol` 157d, `C_get_Variance` 153b, `C_get_varonly` 152b, `C_maxtype` 66, `C_maxtype_pvalue` 70, `C_perm_pvalue` 68, `GE` 22a, `LE` 22a, `LECV` 151b, `P` 25a, `Q` 25e.

⟨ *R_MaximallySelectedTest Prototype* 58 ⟩ ≡

```
SEXP R_MaximallySelectedTest
(
    SEXP LECV,
    SEXP ordered,
    SEXP teststat,
    SEXP minbucket,
    SEXP lower,
    SEXP give_log
)
◇
```

Fragment referenced in 23b, 59.
Uses: `LECV` 151b.

⟨ *R_MaximallySelectedTest* 59 ⟩ ≡

```
⟨ R_MaximallySelectedTest Prototype 58 ⟩
{
    SEXP ans, index, stat, pval, names, permstat;
    int P, mb;

    P = C_get_P(LECV);
    mb = INTEGER(minbucket)[0];

    PROTECT(ans = allocVector(VECSXP, 4));
    PROTECT(names = allocVector(STRSXP, 4));
    SET_VECTOR_ELT(ans, 0, stat = allocVector(REALSXP, 1));
    SET_STRING_ELT(names, 0, mkChar("TestStatistic"));
    SET_VECTOR_ELT(ans, 1, pval = allocVector(REALSXP, 1));
    SET_STRING_ELT(names, 1, mkChar("p.value"));
    SET_VECTOR_ELT(ans, 3, permstat = allocVector(REALSXP, C_get_nresample(LECV)));
    SET_STRING_ELT(names, 3, mkChar("PermutedStatistics"));
    REAL(pval)[0] = NA_REAL;

    if (INTEGER(ordered)[0]) {
        SET_VECTOR_ELT(ans, 2, index = allocVector(INTSXP, 1));
        C_ordered_Xfactor(LECV, mb, INTEGER(teststat)[0],
                          INTEGER(index), REAL(stat), REAL(permstat),
                          REAL(pval), INTEGER(lower)[0],
                          INTEGER(give_log)[0]);
        if (REAL(stat)[0] > 0)
            INTEGER(index)[0]++; /* R style indexing */
    } else {
        SET_VECTOR_ELT(ans, 2, index = allocVector(INTSXP, P));
        C_unordered_Xfactor(LECV, mb, INTEGER(teststat)[0],
                            INTEGER(index), REAL(stat), REAL(permstat),
                            REAL(pval), INTEGER(lower)[0],
                            INTEGER(give_log)[0]);
    }

    SET_STRING_ELT(names, 2, mkChar("index"));
    namesgets(ans, names);

    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 53b.

Uses: `C_get_nresample` 157b, `C_get_P` 151c, `C_ordered_Xfactor` 73, `C_unordered_Xfactor` 78, `LECV` 151b, `P` 25a.

3.6 Test Statistics

⟨ *Test Statistics* 60a ⟩ ≡

⟨ *C_maxstand_Covariance* 60b ⟩
⟨ *C_maxstand_Variance* 61a ⟩
⟨ *C_minstand_Covariance* 61b ⟩
⟨ *C_minstand_Variance* 62a ⟩
⟨ *C_maxabsstand_Covariance* 62b ⟩
⟨ *C_maxabsstand_Variance* 63 ⟩
⟨ *C_quadform* 65 ⟩
⟨ *R_quadform* 64c ⟩
⟨ *C_maxtype* 66 ⟩
⟨ *C_standardise* 67a ⟩
⟨ *C_ordered_Xfactor* 73 ⟩
⟨ *C_unordered_Xfactor* 78 ⟩
◇

Fragment referenced in 24a.

⟨ *C_maxstand_Covariance* 60b ⟩ ≡

```
double C_maxstand_Covariance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *covar_sym,
    const double tol
) {
    double ans = R_NegInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (covar_sym[S(p, p, PQ)] > tol)
            tmp = (linstat[p] - expect[p]) / sqrt(covar_sym[S(p, p, PQ)]);
        if (tmp > ans) ans = tmp;
    }
    return(ans);
}
```
◇

Fragment referenced in 60a.
Defines: `C_maxstand_Covariance` 66.
Uses: `S` 22a.

⟨ *C_maxstand_Variance* 61a ⟩ ≡

```
double C_maxstand_Variance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *var,
    const double tol
) {
    double ans = R_NegInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (var[p] > tol)
            tmp = (linstat[p] - expect[p]) / sqrt(var[p]);
        if (tmp > ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in 60a.
Defines: `C_maxstand_Variance` 66.

⟨ *C_minstand_Covariance* 61b ⟩ ≡

```
double C_minstand_Covariance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *covar_sym,
    const double tol
) {
    double ans = R_PosInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (covar_sym[S(p, p, PQ)] > tol)
            tmp = (linstat[p] - expect[p]) / sqrt(covar_sym[S(p, p, PQ)]);
        if (tmp < ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in 60a.
Defines: `C_minstand_Covariance` 66.
Uses: `S` 22a.

⟨ *C_minstand_Variance* 62a ⟩ ≡

```
double C_minstand_Variance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *var,
    const double tol
) {
    double ans = R_PosInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (var[p] > tol)
            tmp = (linstat[p] - expect[p]) / sqrt(var[p]);
        if (tmp < ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in 60a.
Defines: `C_minstand_Variance` 66.

⟨ *C_maxabsstand_Covariance* 62b ⟩ ≡

```
double C_maxabsstand_Covariance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *covar_sym,
    const double tol
) {
    double ans = R_NegInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (covar_sym[S(p, p, PQ)] > tol)
            tmp = fabs((linstat[p] - expect[p]) /
                  sqrt(covar_sym[S(p, p, PQ)]));
        if (tmp > ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in 60a.
Defines: `C_maxabsstand_Covariance` 66.
Uses: `S` 22a.

⟨ *C_maxabsstand_Variance* 63 ⟩ ≡

```
double C_maxabsstand_Variance
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *var,
    const double tol
) {
    double ans = R_NegInf, tmp = 0.0;

    for (int p = 0; p < PQ; p++) {
        tmp = 0.0;
        if (var[p] > tol)
            tmp = fabs((linstat[p] - expect[p]) / sqrt(var[p]));
        if (tmp > ans) ans = tmp;
    }
    return(ans);
}
◇
```

Fragment referenced in 60a.
Defines: `C_maxabsstand_Variance` 66.

```
> MPinverse <- function(x, tol = sqrt(.Machine$double.eps)) {
+     SVD <- svd(x)
+     pos <- SVD$d > max(tol * SVD$d[1L], 0)
+     inv <- SVD$v[, pos, drop = FALSE] %*%
+              ((1/SVD$d[pos]) * t(SVD$u[, pos, drop = FALSE]))
+     list(MPinv = inv, rank = sum(pos))
+ }
> quadform <- function (linstat, expect, MPinv) {
+     censtat <- linstat - expect
+     censtat %*% MPinv %*% censtat
+ }
> linstat <- ls1$LinearStatistic
> expect <- ls1$Expectation
> MPinv <- MPinverse(vcov(ls1))$MPinv
> MPinv_sym <- MPinv[lower.tri(MPinv, diag = TRUE)]
> qf1 <- quadform(linstat, expect, MPinv)
> qf2 <- .Call(libcoin:::R_quadform, linstat, expect, MPinv_sym)
> stopifnot(isequal(qf1, qf2))
```

"libcoinAPI.h" 64a≡

```
extern SEXP libcoin_R_quadform(
    SEXP linstat, SEXP expect, SEXP MPinv_sym
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_quadform");
    return fun(linstat, expect, MPinv_sym);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: `R_quadform` 64c.

⟨ *R_quadform Prototype* 64b ⟩ ≡

```
SEXP R_quadform
(
    SEXP linstat,
    SEXP expect,
    SEXP MPinv_sym
)
◇
```

Fragment referenced in 23b, 64c.
Uses: `R_quadform` 64c.

⟨ *R_quadform* 64c ⟩ ≡

```
⟨ R_quadform Prototype 64b ⟩
{
    SEXP ans;
    int n, PQ;
    double *dlinstat, *dexpect, *dMPinv_sym, *dans;

    n = NCOL(linstat);
    PQ = NROW(linstat);
    dlinstat = REAL(linstat);
    dexpect = REAL(expect);
    dMPinv_sym = REAL(MPinv_sym);

    PROTECT(ans = allocVector(REALSXP, n));
    dans = REAL(ans);
    for (int i = 0; i < n; i++)
      dans[i] = C_quadform(PQ, dlinstat + PQ * i, dexpect, dMPinv_sym);

    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 60a.
Defines: `R_quadform` 64ab, 164, 165.
Uses: `C_quadform` 65, `NCOL` 139c, `NROW` 139b.

⟨ *C_quadform* 65 ⟩ ≡

```
double C_quadform
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *MPinv_sym
) {
    double ans = 0.0, tmp = 0.0;

    for (int q = 0; q < PQ; q++) {
        tmp = 0.0;
        for (int p = 0; p < PQ; p++)
            tmp += (linstat[p] - expect[p]) * MPinv_sym[S(p, q, PQ)];
        ans += tmp * (linstat[q] - expect[q]);
    }

    return(ans);
}
◇
```

Fragment referenced in 60a.
Defines: `C_quadform` 55, 64c, 76c.
Uses: `S` 22a.

⟨ *C_maxtype* 66 ⟩ ≡

```
double C_maxtype
(
    const int PQ,
    const double *linstat,
    const double *expect,
    const double *covar,
    const int varonly,
    const double tol,
    const int alternative
) {
    double ret = 0.0;

    if (varonly) {
        if (alternative == ALTERNATIVE_twosided) {
            ret = C_maxabsstand_Variance(PQ, linstat, expect, covar, tol);
        } else if (alternative == ALTERNATIVE_less) {
            ret = C_minstand_Variance(PQ, linstat, expect, covar, tol);
        } else if (alternative == ALTERNATIVE_greater) {
            ret = C_maxstand_Variance(PQ, linstat, expect, covar, tol);
        }
    } else {
        if (alternative == ALTERNATIVE_twosided) {
            ret = C_maxabsstand_Covariance(PQ, linstat, expect, covar, tol);
        } else if (alternative == ALTERNATIVE_less) {
            ret = C_minstand_Covariance(PQ, linstat, expect, covar, tol);
        } else if (alternative == ALTERNATIVE_greater) {
            ret = C_maxstand_Covariance(PQ, linstat, expect, covar, tol);
        }
    }
    return(ret);
}
```
◇

Fragment referenced in 60a.
Defines: `C_maxtype` 57, 76c.
Uses: `C_maxabsstand_Covariance` 62b, `C_maxabsstand_Variance` 63, `C_maxstand_Covariance` 60b, `C_maxstand_Variance` 61a, `C_minstand_Covariance` 61b, `C_minstand_Variance` 62a.

⟨ *C_standardise* 67a ⟩ ≡

```
void C_standardise
(
    const int PQ,
    double *linstat,            /* in place standardisation */
    const double *expect,
    const double *covar,
    const int varonly,
    const double tol
) {
    double var;

    for (int p = 0; p < PQ; p++) {
        if (varonly) {
            var = covar[p];
        } else {
            var = covar[S(p, p, PQ)];
        }
        if (var > tol) {
            linstat[p] = (linstat[p] - expect[p]) / sqrt(var);
        } else {
            linstat[p] = NAN;
        }
    }
}
◇
```

Fragment referenced in 60a.
Defines: `C_standardise` 42a.
Uses: `S` 22a.

⟨ *P-Values* 67b ⟩ ≡

⟨ *C_chisq_pvalue* 67c ⟩
⟨ *C_perm_pvalue* 68 ⟩
⟨ *C_norm_pvalue* 69 ⟩
⟨ *C_maxtype_pvalue* 70 ⟩
◇

Fragment referenced in 24a.

⟨ *C_chisq_pvalue* 67c ⟩ ≡

```
/* lower = 1 means p-value, lower = 0 means 1 - p-value */
double C_chisq_pvalue
(
    const double stat,
    const int df,
    const int lower,
    const int give_log
) {
    return(pchisq(stat, (double) df, lower, give_log));
}
◇
```

Fragment referenced in 67b.
Defines: `C_chisq_pvalue` 55.

⟨ *C_perm_pvalue* 68 ⟩ ≡

```
double C_perm_pvalue
(
    const int greater,
    const double nresample,
    const int lower,
    const int give_log
) {
    double ret;

    if (give_log) {
         if (lower) {
             ret = log1p(- (double) greater / nresample);
         } else {
             ret = log(greater) - log(nresample);
         }
    } else {
        if (lower) {
            ret = 1.0 - (double) greater / nresample;
        } else {
            ret = (double) greater / nresample;
        }
    }
    return(ret);
}
◇
```

Fragment referenced in 67b.
Defines: `C_perm_pvalue` 55, 57, 77.

⟨ *C_norm_pvalue* 69 ⟩ ≡

```
double C_norm_pvalue
(
    const double stat,
    const int alternative,
    const int lower,
    const int give_log
) {
    double ret;

    if (alternative == ALTERNATIVE_less) {
        return(pnorm(stat, 0.0, 1.0, 1 - lower, give_log));
    } else if (alternative == ALTERNATIVE_greater) {
        return(pnorm(stat, 0.0, 1.0, lower, give_log));
    } else if (alternative == ALTERNATIVE_twosided) {
        if (lower) {
            ret = pnorm(fabs(stat)*-1.0, 0.0, 1.0, 1, 0);
            if (give_log) {
                return(log1p(- 2 * ret));
            } else {
                return(1 - 2 * ret);
            }
        } else {
            ret = pnorm(fabs(stat)*-1.0, 0.0, 1.0, 1, give_log);
            if (give_log) {
                return(ret + log(2));
            } else {
                return(2 * ret);
            }
        }
    }
    return(NA_REAL);
}
```
◇

Fragment referenced in 67b.

⟨ *C_maxtype_pvalue* 70 ⟩ ≡

```
double C_maxtype_pvalue
(
    const double stat,
    const double *Covariance,
    const int n,
    const int alternative,
    const int lower,
    const int give_log,
    int maxpts, /* const? */
    double releps,
    double abseps,
    double tol
) {
    int nu = 0, inform, i, j, sub, nonzero, *infin, *index, rnd = 0;
    double ans, myerror, *lowerbnd, *upperbnd, *delta, *corr, *sd;

    /* univariate problem */
    if (n == 1)
        return(C_norm_pvalue(stat, alternative, lower, give_log));

    ⟨ Setup mvtnorm Memory 71 ⟩

    ⟨ Setup mvtnorm Correlation 72a ⟩

    /* call mvtnorm's mvtdst C function defined in mvtnorm/include/mvtnormAPI.h */
    mvtnorm_C_mvtdst(&nonzero, &nu, lowerbnd, upperbnd, infin, corr, delta,
                     &maxpts, &abseps, &releps, &myerror, &ans, &inform, &rnd);

    /* inform == 0 means: everything is OK */
    switch (inform) {
        case 0: break;
        case 1: warning("cmvnorm: completion with ERROR > EPS"); break;
        case 2: warning("cmvnorm: N > 1000 or N < 1");
                ans = 0.0;
                break;
        case 3: warning("cmvnorm: correlation matrix not positive semi-definite");
                ans = 0.0;
                break;
        default: warning("cmvnorm: unknown problem in MVTDST");
                ans = 0.0;
    }
    Free(corr); Free(sd); Free(lowerbnd); Free(upperbnd);
    Free(infin); Free(delta); Free(index);

    /* ans = 1 - p-value */
    if (lower) {
        if (give_log)
            return(log(ans)); /* log(1 - p-value) */
        return(ans); /* 1 - p-value */
    } else {
        if (give_log)
            return(log1p(ans)); /* log(p-value) */
        return(1 - ans); /* p-value */
    }
}
◇
```

Fragment referenced in 67b.
Defines: `C_maxtype_pvalue` 57.
Uses: `N` 24bc.

⟨ *Setup mvtnorm Memory* 71 ⟩ ≡

```
if (n == 2)
    corr = Calloc(1, double);
else
    corr = Calloc(n + ((n - 2) * (n - 1))/2, double);

sd = Calloc(n, double);
lowerbnd = Calloc(n, double);
upperbnd = Calloc(n, double);
infin = Calloc(n, int);
delta = Calloc(n, double);
index = Calloc(n, int);

/* determine elements with non-zero variance */

nonzero = 0;
for (i = 0; i < n; i++) {
    if (Covariance[S(i, i, n)] > tol) {
        index[nonzero] = i;
        nonzero++;
    }
}
◇
```

Fragment referenced in 70.
Uses: `S` 22a.

`mvtdst` assumes the unique elements of the triangular covariance matrix to be passed as argument `CORREL`

⟨ *Setup mvtnorm Correlation* 72a ⟩ ≡

```
for (int nz = 0; nz < nonzero; nz++) {
    /* handle elements with non-zero variance only */
    i = index[nz];

    /* standard deviations */
    sd[i] = sqrt(Covariance[S(i, i, n)]);

    if (alternative == ALTERNATIVE_less) {
        lowerbnd[nz] = stat;
        upperbnd[nz] = R_PosInf;
        infin[nz] = 1;
    } else if (alternative == ALTERNATIVE_greater) {
        lowerbnd[nz] = R_NegInf;
        upperbnd[nz] = stat;
        infin[nz] = 0;
    } else if (alternative == ALTERNATIVE_twosided) {
        lowerbnd[nz] = fabs(stat) * -1.0;
        upperbnd[nz] = fabs(stat);
        infin[nz] = 2;
    }

    delta[nz] = 0.0;

    /* set up vector of correlations, i.e., the upper
       triangular part of the covariance matrix) */
    for (int jz = 0; jz < nz; jz++) {
        j = index[jz];
        sub = (int) (jz + 1) + (double) ((nz - 1) * nz) / 2 - 1;
        if (sd[i] == 0.0 || sd[j] == 0.0)
            corr[sub] = 0.0;
        else
            corr[sub] = Covariance[S(i, j, n)] / (sd[i] * sd[j]);
    }
}
◇
```

Fragment referenced in 70.
Uses: `S` 22a.

⟨ *maxstat Xfactor Variables* 72b ⟩ ≡

```
SEXP LECV,
const int minbucket,
const int teststat,
int *wmax,
double *maxstat,
double *bmaxstat,
double *pval,
const int lower,
const int give_log
◇
```

Fragment referenced in 73, 78.
Uses: `LECV` 151b.

⟨ *C_ordered_Xfactor* 73 ⟩ ≡

```
void C_ordered_Xfactor
(
    ⟨ maxstat Xfactor Variables 72b ⟩
) {
    ⟨ Setup maxstat Variables 74 ⟩

    ⟨ Setup maxstat Memory 75 ⟩

    wmax[0] = NA_INTEGER;

    for (int p = 0; p < P; p++) {
        sumleft += ExpX[p];
        sumright -= ExpX[p];

        for (int q = 0; q < Q; q++) {
            mlinstat[q] += linstat[q * P + p];
            for (R_xlen_t np = 0; np < nresample; np++)
                mblinstat[q + np * Q] += blinstat[q * P + p + np * PQ];
            mexpect[q] += expect[q * P + p];
            if (B == 1) {
                ⟨ Compute maxstat Variance / Covariance Directly 76b ⟩
            } else {
                ⟨ Compute maxstat Variance / Covariance from Total Covariance 76a ⟩
            }
        }

        if ((sumleft >= minbucket) && (sumright >= minbucket) && (ExpX[p] > 0)) {
            ls = mlinstat;
            /* compute MPinv only once */
            if (teststat != TESTSTAT_maximum)
                C_MPinv_sym(mcovar, Q, tol, mMPinv, &rank);
            ⟨ Compute maxstat Test Statistic 76c ⟩
            if (tmp > maxstat[0]) {
                wmax[0] = p;
                maxstat[0] = tmp;
            }

            for (R_xlen_t np = 0; np < nresample; np++) {
                ls = mblinstat + np * Q;
                ⟨ Compute maxstat Test Statistic 76c ⟩
                if (tmp > bmaxstat[np])
                    bmaxstat[np] = tmp;
            }
        }
    }
    ⟨ Compute maxstat Permutation P-Value 77 ⟩
    Free(mlinstat); Free(mexpect); Free(mblinstat);
    Free(mvar); Free(mcovar); Free(mMPinv);
    if (nresample == 0) Free(blinstat);
}
◇
```

Fragment referenced in 60a.
Defines: `C_ordered_Xfactor` 37b, 47, 59.
Uses: `B` 28c, `P` 25a, `Q` 25e.

⟨*Setup maxstat Variables* 74⟩ ≡

```
double *linstat, *expect, *covar, *varinf, *covinf, *ExpX, *blinstat, tol, *ls;
int P, Q, B;
R_xlen_t nresample;

double *mlinstat, *mblinstat, *mexpect, *mvar, *mcovar, *mMPinv,
       tmp, sumleft, sumright, sumweights;
int rank, PQ, greater;

Q = C_get_Q(LECV);
P = C_get_P(LECV);
PQ = mPQB(P, Q, 1);
B = C_get_B(LECV);
if (B > 1) {
    if (C_get_varonly(LECV))
        error("need covarinance for maximally statistics with blocks");
    covar = C_get_Covariance(LECV);
} else {
    covar = C_get_Variance(LECV); /* make -Wall happy */
}
linstat = C_get_LinearStatistic(LECV);
expect = C_get_Expectation(LECV);
ExpX = C_get_ExpectationX(LECV);
/* both need to be there */
varinf = C_get_VarianceInfluence(LECV);
covinf = C_get_CovarianceInfluence(LECV);
nresample = C_get_nresample(LECV);
if (nresample > 0)
    blinstat = C_get_PermutedLinearStatistic(LECV);
tol = C_get_tol(LECV);
◇
```

Fragment referenced in 73, 78.
Uses: `B` 28c, `C_get_B` 157a, `C_get_Covariance` 154a, `C_get_CovarianceInfluence` 155a, `C_get_Expectation` 153a, `C_get_ExpectationX` 154b, `C_get_LinearStatistic` 152d, `C_get_nresample` 157b, `C_get_P` 151c, `C_get_PermutedLinearStatistic` 157c, `C_get_Q` 152a, `C_get_tol` 157d, `C_get_Variance` 153b, `C_get_VarianceInfluence` 155b, `C_get_varonly` 152b, `LECV` 151b, `mPQB` 141a, `P` 25a, `Q` 25e, `sumweights` 27a.

⟨ *Setup maxstat Memory* 75 ⟩ ≡

```
mlinstat = Calloc(Q, double);
mexpect = Calloc(Q, double);
if (teststat == TESTSTAT_maximum) {
   mvar = Calloc(Q, double);
   /* not needed, but allocate anyway to make -Wmaybe-uninitialized happy */
   mcovar = Calloc(1, double);
   mMPinv = Calloc(1, double);
} else {
   mcovar = Calloc(Q * (Q + 1) / 2, double);
   mMPinv = Calloc(Q * (Q + 1) / 2, double);
   /* not needed, but allocate anyway to make -Wmaybe-uninitialized happy */
   mvar = Calloc(1, double);
}
if (nresample > 0) {
    mblinstat = Calloc(Q * nresample, double);
} else { /* not needed, but allocate anyway to make -Wmaybe-uninitialized happy */
    mblinstat = Calloc(1, double);
    blinstat = Calloc(1, double);
}

maxstat[0] = 0.0;

for (int q = 0; q < Q; q++) {
    mlinstat[q] = 0.0;
    mexpect[q] = 0.0;
    if (teststat == TESTSTAT_maximum)
        mvar[q] = 0.0;
    for (R_xlen_t np = 0; np < nresample; np++) {
        mblinstat[q + np * Q] = 0.0;
        bmaxstat[np] = 0.0;
    }
}
if (teststat == TESTSTAT_quadratic) {
    for (int q = 0; q < Q * (Q + 1) / 2; q++)
        mcovar[q] = 0.0;
}

sumleft = 0.0;
sumright = 0.0;
for (int p = 0; p < P; p++)
    sumright += ExpX[p];
sumweights = sumright;
◇
```

Fragment referenced in 73, 78.
Uses: `P` 25a, `Q` 25e, `sumweights` 27a.

⟨ *Compute maxstat Variance / Covariance from Total Covariance* 76a ⟩ ≡

```
if (teststat == TESTSTAT_maximum) {
    for (int pp = 0; pp < p; pp++)
        mvar[q] += 2 * covar[S(pp + q * P, p + P * q, mPQB(P, Q, 1))];
     mvar[q] += covar[S(p + q * P, p + P * q, mPQB(P, Q, 1))];
} else {
     for (int qq = 0; qq <= q; qq++) {
         for (int pp = 0; pp < p; pp++)
             mcovar[S(q, qq, Q)] += 2 * covar[S(pp + q * P, p + P * qq, mPQB(P, Q, 1))];
         mcovar[S(q, qq, Q)] += covar[S(p + q * P, p + P * qq, mPQB(P, Q, 1))];
     }
}
```
◇

Fragment referenced in 73.
Uses: `mPQB` 141a, `P` 25a, `Q` 25e, `S` 22a.

⟨ *Compute maxstat Variance / Covariance Directly* 76b ⟩ ≡

```
/* does not work with blocks! */
if (teststat == TESTSTAT_maximum) {
    C_VarianceLinearStatistic(1, Q, varinf, &sumleft, &sumleft,
                              sumweights, 0, mvar);
} else {
    C_CovarianceLinearStatistic(1, Q, covinf, &sumleft, &sumleft,
                                sumweights, 0, mcovar);
}
```
◇

Fragment referenced in 73.
Uses: `C_CovarianceLinearStatistic` 83, `C_VarianceLinearStatistic` 84, `Q` 25e, `sumweights` 27a.

⟨ *Compute maxstat Test Statistic* 76c ⟩ ≡

```
if (teststat == TESTSTAT_maximum) {
    tmp = C_maxtype(Q, ls, mexpect, mvar, 1, tol,
                    ALTERNATIVE_twosided);
} else {
    tmp = C_quadform(Q, ls, mexpect, mMPinv);
}
```
◇

Fragment referenced in 73, 78.
Uses: `C_maxtype` 66, `C_quadform` 65, `Q` 25e.

⟨ *Compute maxstat Permutation P-Value* 77 ⟩ ≡

```
if (nresample > 0) {
    greater = 0;
    for (R_xlen_t np = 0; np < nresample; np++) {
        if (bmaxstat[np] > maxstat[0]) greater++;
    }
    pval[0] = C_perm_pvalue(greater, nresample, lower, give_log);
}
◇
```

Fragment referenced in 73, 78.
Uses: `C_perm_pvalue` 68.

⟨ *C_unordered_Xfactor* 78 ⟩ ≡

```
void C_unordered_Xfactor
(
    ⟨ maxstat Xfactor Variables 72b ⟩
) {
    double *mtmp;
    int qPp, nc, *levels, Pnonzero, *indl, *contrast;

    ⟨ Setup maxstat Variables 74 ⟩

    ⟨ Setup maxstat Memory 75 ⟩
    mtmp = Calloc(P, double);

    for (int p = 0; p < P; p++) wmax[p] = NA_INTEGER;

    ⟨ Count Levels 79a ⟩

    for (int j = 1; j < mi; j++) { /* go though all splits */

        ⟨ Setup unordered maxstat Contrasts 79b ⟩

        ⟨ Compute unordered maxstat Linear Statistic and Expectation 80a ⟩

        if (B == 1) {
            ⟨ Compute unordered maxstat Variance / Covariance Directly 81a ⟩
        } else {
            ⟨ Compute unordered maxstat Variance / Covariance from Total Covariance 80b ⟩
        }

        if ((sumleft >= minbucket) && (sumright >= minbucket)) {
            ls = mlinstat;
            /* compute MPinv only once */
            if (teststat != TESTSTAT_maximum)
                C_MPinv_sym(mcovar, Q, tol, mMPinv, &rank);
            ⟨ Compute maxstat Test Statistic 76c ⟩
            if (tmp > maxstat[0]) {
                for (int p = 0; p < Pnonzero; p++)
                    wmax[levels[p]] = contrast[levels[p]];
                maxstat[0] = tmp;
            }

            for (R_xlen_t np = 0; np < nresample; np++) {
                ls = mblinstat + np * Q;
                ⟨ Compute maxstat Test Statistic 76c ⟩
                if (tmp > bmaxstat[np])
                    bmaxstat[np] = tmp;
            }
        }
    }

    ⟨ Compute maxstat Permutation P-Value 77 ⟩

    Free(mlinstat); Free(mexpect); Free(levels); Free(contrast); Free(indl); Free(mtmp);
    Free(mblinstat); Free(mvar); Free(mcovar); Free(mMPinv);
    if (nresample == 0) Free(blinstat);
}
◇
```

Fragment referenced in 60a.
Defines: `C_unordered_Xfactor` 37b, 59.
Uses: `B` 28c, `P` 25a, `Q` 25e.

⟨ *Count Levels* 79a ⟩ ≡

```
contrast = Calloc(P, int);
Pnonzero = 0;
for (int p = 0; p < P; p++) {
    if (ExpX[p] > 0) Pnonzero++;
}
levels = Calloc(Pnonzero, int);
nc = 0;
for (int p = 0; p < P; p++) {
    if (ExpX[p] > 0) {
        levels[nc] = p;
        nc++;
    }
}

if (Pnonzero >= 31)
    error("cannot search for unordered splits in >= 31 levels");

int mi = 1;
for (int l = 1; l < Pnonzero; l++) mi *= 2;
indl = Calloc(Pnonzero, int);
for (int p = 0; p < Pnonzero; p++) indl[p] = 0;
◇
```

Fragment referenced in 78.
Uses: P 25a.

⟨ *Setup unordered maxstat Contrasts* 79b ⟩ ≡

```
/* indl determines if level p is left or right */
int jj = j;
for (int l = 1; l < Pnonzero; l++) {
    indl[l] = (jj%2);
    jj /= 2;
}

sumleft = 0.0;
sumright = 0.0;
for (int p = 0; p < P; p++) contrast[p] = 0;
for (int p = 0; p < Pnonzero; p++) {
    sumleft += indl[p] * ExpX[levels[p]];
    sumright += (1 - indl[p]) * ExpX[levels[p]];
    contrast[levels[p]] = indl[p];
}
◇
```

Fragment referenced in 78.
Uses: P 25a.

⟨ *Compute unordered maxstat Linear Statistic and Expectation* 80a ⟩ ≡

```
for (int q = 0; q < Q; q++) {
    mlinstat[q] = 0.0;
    mexpect[q] = 0.0;
    for (R_xlen_t np = 0; np < nresample; np++)
        mblinstat[q + np * Q] = 0.0;
    for (int p = 0; p < P; p++) {
        qPp = q * P + p;
        mlinstat[q] += contrast[p] * linstat[qPp];
        mexpect[q] += contrast[p] * expect[qPp];
        for (R_xlen_t np = 0; np < nresample; np++)
            mblinstat[q + np * Q] += contrast[p] * blinstat[q * P + p + np * PQ];
    }
}
◇
```

Fragment referenced in 78.
Uses: `P` 25a, `Q` 25e.

⟨ *Compute unordered maxstat Variance / Covariance from Total Covariance* 80b ⟩ ≡

```
if (teststat == TESTSTAT_maximum) {
    for (int q = 0; q < Q; q++) {
        mvar[q] = 0.0;
        for (int p = 0; p < P; p++) {
            qPp = q * P + p;
            mtmp[p] = 0.0;
            for (int pp = 0; pp < P; pp++)
                mtmp[p] += contrast[pp] * covar[S(pp + q * P, qPp, PQ)];
        }
        for (int p = 0; p < P; p++)
            mvar[q] += contrast[p] * mtmp[p];
    }
} else {
    for (int q = 0; q < Q; q++) {
        for (int qq = 0; qq <= q; qq++)
            mcovar[S(q, qq, Q)] = 0.0;
        for (int qq = 0; qq <= q; qq++) {
            for (int p = 0; p < P; p++) {
                mtmp[p] = 0.0;
                for (int pp = 0; pp < P; pp++)
                    mtmp[p] += contrast[pp] * covar[S(pp + q * P, p + P * qq,
                                                      mPQB(P, Q, 1))];
            }
            for (int p = 0; p < P; p++)
                mcovar[S(q, qq, Q)] += contrast[p] * mtmp[p];
        }
    }
}
◇
```

Fragment referenced in 78.
Uses: `mPQB` 141a, `P` 25a, `Q` 25e, `S` 22a.

⟨ *Compute unordered maxstat Variance / Covariance Directly* 81a ⟩ ≡

```
if (teststat == TESTSTAT_maximum) {
    C_VarianceLinearStatistic(1, Q, varinf, &sumleft, &sumleft,
                              sumweights, 0, mvar);
} else {
    C_CovarianceLinearStatistic(1, Q, covinf, &sumleft, &sumleft,
                                sumweights, 0, mcovar);
}
◇
```

Fragment referenced in 78.
Uses: `C_CovarianceLinearStatistic` 83, `C_VarianceLinearStatistic` 84, `Q` 25e, `sumweights` 27a.

3.7 Linear Statistics

⟨ *LinearStatistics* 81b ⟩ ≡

```
⟨ RC_LinearStatistic 81d ⟩
◇
```

Fragment referenced in 24a.

⟨ *RC_LinearStatistic Prototype* 81c ⟩ ≡

```
void RC_LinearStatistic
(
    ⟨ R x Input 24d ⟩
    ⟨ C integer N Input 24c ⟩,
    ⟨ C integer P Input 25a ⟩,
    ⟨ C real y Input 26a ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    ⟨ C KronSums Answer 101d ⟩
)
◇
```

Fragment referenced in 81d.
Uses: `RC_LinearStatistic` 81d.

⟨ *RC_LinearStatistic* 81d ⟩ ≡

```
⟨ RC_LinearStatistic Prototype 81c ⟩
{
    double center;

    RC_KronSums(x, N, P, y, Q, !DoSymmetric, &center, &center, !DoCenter, weights,
                subset, offset, Nsubset, PQ_ans);
}
◇
```

Fragment referenced in 81b.
Defines: `RC_LinearStatistic` 35b, 81c.
Uses: `DoCenter` 22b, `DoSymmetric` 22b, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `Q` 25e, `RC_KronSums` 101a, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc, `y` 25d, 26ab.

3.8 Expectation and Covariance

⟨ *ExpectationCovariances* 82a ⟩ ≡

> ⟨ *RC_ExpectationInfluence* 86a ⟩
> ⟨ *R_ExpectationInfluence* 85b ⟩
> ⟨ *RC_CovarianceInfluence* 88a ⟩
> ⟨ *R_CovarianceInfluence* 87a ⟩
> ⟨ *RC_ExpectationX* 90 ⟩
> ⟨ *R_ExpectationX* 89a ⟩
> ⟨ *RC_CovarianceX* 93a ⟩
> ⟨ *R_CovarianceX* 92a ⟩
> ⟨ *C_ExpectationLinearStatistic* 82b ⟩
> ⟨ *C_CovarianceLinearStatistic* 83 ⟩
> ⟨ *C_VarianceLinearStatistic* 84 ⟩
> ◇

Fragment referenced in 24a.

3.8.1 Linear Statistic

⟨ *C_ExpectationLinearStatistic* 82b ⟩ ≡

```
void C_ExpectationLinearStatistic
(
    ⟨ C integer P Input 25a ⟩,
    ⟨ C integer Q Input 25e ⟩,
    double *ExpInf,
    double *ExpX,
    const int add,
    double *PQ_ans
) {
    if (!add)
        for (int p = 0; p < mPQB(P, Q, 1); p++) PQ_ans[p] = 0.0;

    for (int p = 0; p < P; p++) {
        for (int q = 0; q < Q; q++)
            PQ_ans[q * P + p] += ExpX[p] * ExpInf[q];
    }
}
◇
```

Fragment referenced in 82a.
Defines: `C_ExpectationLinearStatistic` 37a, 46c.
Uses: `mPQB` 141a, `P` 25a, `Q` 25e.

⟨ *C_CovarianceLinearStatistic* 83 ⟩ ≡

```
void C_CovarianceLinearStatistic
(
    ⟨ C integer P Input 25a ⟩,
    ⟨ C integer Q Input 25e ⟩,
    double *CovInf,
    double *ExpX,
    double *CovX,
    ⟨ C sumweights Input 27a ⟩,
    const int add,
    double *PQPQ_sym_ans
) {
    double f1 = sumweights / (sumweights - 1);
    double f2 = 1.0 / (sumweights - 1);
    double tmp, *PP_sym_tmp;

    if (mPQB(P, Q, 1) == 1) {
        tmp = f1 * CovInf[0] * CovX[0];
        tmp -= f2 * CovInf[0] * ExpX[0] * ExpX[0];
        if (add) {
            PQPQ_sym_ans[0] += tmp;
        } else {
            PQPQ_sym_ans[0] = tmp;
        }
    } else {
        PP_sym_tmp = Calloc(PP12(P), double);
        C_KronSums_sym_(ExpX, 1, P,
                        PP_sym_tmp);
        for (int p = 0; p < PP12(P); p++)
            PP_sym_tmp[p] = f1 * CovX[p] - f2 * PP_sym_tmp[p];
        C_kronecker_sym(CovInf, Q, PP_sym_tmp, P, 1 - (add >= 1),
                        PQPQ_sym_ans);
        Free(PP_sym_tmp);
    }
}
◇
```

Fragment referenced in 82a.
Defines: `C_CovarianceLinearStatistic` 38a, 47, 76b, 81a, 84.
Uses: `C_kronecker_sym` 144, `mPQB` 141a, `P` 25a, `PP12` 140b, `Q` 25e, `sumweights` 27a.

⟨ *C_VarianceLinearStatistic* 84 ⟩ ≡

```
void C_VarianceLinearStatistic
(
    ⟨ C integer P Input 25a ⟩,
    ⟨ C integer Q Input 25e ⟩,
    double *VarInf,
    double *ExpX,
    double *VarX,
    ⟨ C sumweights Input 27a ⟩,
    const int add,
    double *PQ_ans
) {
    if (mPQB(P, Q, 1) == 1) {
        C_CovarianceLinearStatistic(P, Q, VarInf, ExpX, VarX,
                                    sumweights, (add >= 1),
                                    PQ_ans);
    } else {
        double *P_tmp;
        P_tmp = Calloc(P, double);
        double f1 = sumweights / (sumweights - 1);
        double f2 = 1.0 / (sumweights - 1);
        for (int p = 0; p < P; p++)
            P_tmp[p] = f1 * VarX[p] - f2 * ExpX[p] * ExpX[p];
        C_kronecker(VarInf, 1, Q, P_tmp, 1, P, 1 - (add >= 1),
                    PQ_ans);
        Free(P_tmp);
    }
}
◇
```

Fragment referenced in 82a.
Defines: `C_VarianceLinearStatistic` 37c, 47, 76b, 81a.
Uses: `C_CovarianceLinearStatistic` 83, `C_kronecker` 143, `mPQB` 141a, `P` 25a, `Q` 25e, `sumweights` 27a.

3.8.2 Influence

```
> sumweights <- sum(weights[subset])
> expecty <- a0 <- colSums(y[subset, ] * weights[subset]) / sumweights
> a1 <- .Call(libcoin:::R_ExpectationInfluence, y, weights, subset);
> a2 <- .Call(libcoin:::R_ExpectationInfluence, y, as.double(weights), as.double(subset));
> a3 <- .Call(libcoin:::R_ExpectationInfluence, y, weights, as.double(subset));
> a4 <- .Call(libcoin:::R_ExpectationInfluence, y, as.double(weights), subset);
> a5 <- LinStatExpCov(x, y, weights = weights, subset = subset)$ExpectationInfluence
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4) &&
+           isequal(a0, a5))
```

⟨ *R_ExpectationInfluence Prototype* 85a ⟩ ≡

```
SEXP R_ExpectationInfluence
(
    ⟨ R y Input 25d ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩
)
◇
```

Fragment referenced in 23b, 85b.
Uses: `R_ExpectationInfluence` 85b.

⟨ *R_ExpectationInfluence* 85b ⟩ ≡

```
⟨ R_ExpectationInfluence Prototype 85a ⟩
{
    SEXP ans;
    ⟨ C integer Q Input 25e ⟩;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;
    double sumweights;

    Q = NCOL(y);
    N = XLENGTH(y) / Q;
    Nsubset = XLENGTH(subset);

    sumweights = RC_Sums(N, weights, subset, Offset0, Nsubset);

    PROTECT(ans = allocVector(REALSXP, Q));
    RC_ExpectationInfluence(N, y, Q, weights, subset, Offset0, Nsubset, sumweights, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 82a.
Defines: `R_ExpectationInfluence` 85a, 87a, 164, 165.
Uses: `N` 24bc, `NCOL` 139c, `Nsubset` 27c, `Offset0` 22b, `Q` 25e, `RC_ExpectationInfluence` 86a, `RC_Sums` 96a, `subset` 27be, 28a, `sumweights` 27a, `weights` 26c, `weights`, 26de, `y` 25d, 26ab.

⟨ *RC_ExpectationInfluence Prototype* 85c ⟩ ≡

```
void RC_ExpectationInfluence
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ R y Input 25d ⟩
    ⟨ C integer Q Input 25e ⟩,
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    ⟨ C sumweights Input 27a ⟩,
    ⟨ C colSums Answer 114c ⟩
)
◇
```

Fragment referenced in 86a.
Uses: `RC_ExpectationInfluence` 86a.

⟨ *RC_ExpectationInfluence* 86a ⟩ ≡

```
⟨ RC_ExpectationInfluence Prototype 85c ⟩
{
    double center;

    RC_colSums(REAL(y), N, Q, Power1, &center, !DoCenter, weights,
               subset, offset, Nsubset, P_ans);
    for (int q = 0; q < Q; q++)
        P_ans[q] = P_ans[q] / sumweights;
}
◇
```

Fragment referenced in 82a.
Defines: `RC_ExpectationInfluence` 37a, 46c, 85bc.
Uses: `DoCenter` 22b, `N` 24bc, `Nsubset` 27c, `offset` 27d, `Power1` 22b, `Q` 25e, `RC_colSums` 114a, `subset` 27be, 28a, `sumweights` 27a, `weights` 26c, `weights,` 26de, `y` 25d, 26ab.

```
> sumweights <- sum(weights[subset])
> yc <- t(t(y) - expecty)
> r1y <- rep(1:ncol(y), ncol(y))
> r2y <- rep(1:ncol(y), each = ncol(y))
> a0 <- colSums(yc[subset, r1y] * yc[subset, r2y] * weights[subset]) / sumweights
> a0 <- matrix(a0, ncol = ncol(y))
> vary <- diag(a0)
> a0 <- a0[lower.tri(a0, diag = TRUE)]
> a1 <- .Call(libcoin:::R_CovarianceInfluence, y, weights, subset, 0L);
> a2 <- .Call(libcoin:::R_CovarianceInfluence, y, as.double(weights), as.double(subset), 0L);
> a3 <- .Call(libcoin:::R_CovarianceInfluence, y, weights, as.double(subset), 0L);
> a4 <- .Call(libcoin:::R_CovarianceInfluence, y, as.double(weights), subset, 0L);
> a5 <- LinStatExpCov(x, y, weights = weights, subset = subset)$CovarianceInfluence
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4) &&
+           isequal(a0, a5))
> a1 <- .Call(libcoin:::R_CovarianceInfluence, y, weights, subset, 1L);
> a2 <- .Call(libcoin:::R_CovarianceInfluence, y, as.double(weights), as.double(subset), 1L);
> a3 <- .Call(libcoin:::R_CovarianceInfluence, y, weights, as.double(subset), 1L);
> a4 <- .Call(libcoin:::R_CovarianceInfluence, y, as.double(weights), subset, 1L);
> a5 <- LinStatExpCov(x, y, weights = weights, subset = subset, varonly = TRUE)$VarianceInfluence
> a0 <- vary
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4) &&
+           isequal(a0, a5))
```

⟨ *R_CovarianceInfluence Prototype* 86b ⟩ ≡

```
SEXP R_CovarianceInfluence
(
    ⟨ R y Input 25d ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    SEXP varonly
)
◇
```

Fragment referenced in 23b, 87a.
Uses: `R_CovarianceInfluence` 87a.

⟨ *R_CovarianceInfluence* 87a ⟩ ≡

```
⟨ R_CovarianceInfluence Prototype 86b ⟩
{
    SEXP ans;
    SEXP ExpInf;
    ⟨ C integer Q Input 25e ⟩;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;
    double sumweights;

    Q = NCOL(y);
    N = XLENGTH(y) / Q;
    Nsubset = XLENGTH(subset);

    PROTECT(ExpInf = R_ExpectationInfluence(y, weights, subset));

    sumweights = RC_Sums(N, weights, subset, Offset0, Nsubset);

    if (INTEGER(varonly)[0]) {
        PROTECT(ans = allocVector(REALSXP, Q));
    } else {
        PROTECT(ans = allocVector(REALSXP, Q * (Q + 1) / 2));
    }
    RC_CovarianceInfluence(N, y, Q, weights, subset, Offset0, Nsubset, REAL(ExpInf), sumweights,
                           INTEGER(varonly)[0], REAL(ans));
    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 82a.
Defines: `R_CovarianceInfluence` 86b, 164, 165.
Uses: `N` 24bc, `NCOL` 139c, `Nsubset` 27c, `Offset0` 22b, `Q` 25e, `RC_CovarianceInfluence` 88a, `RC_Sums` 96a, `R_ExpectationInfluence` 85b, `subset` 27be, 28a, `sumweights` 27a, `weights` 26c, `weights,` 26de, `y` 25d, 26ab.

⟨ *RC_CovarianceInfluence Prototype* 87b ⟩ ≡

```
void RC_CovarianceInfluence
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ R y Input 25d ⟩
    ⟨ C integer Q Input 25e ⟩,
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    double *ExpInf,
    ⟨ C sumweights Input 27a ⟩,
    int VARONLY,
    ⟨ C KronSums Answer 101d ⟩
)
◇
```

Fragment referenced in 88a.
Uses: `RC_CovarianceInfluence` 88a.

⟨ *RC_CovarianceInfluence* 88a ⟩ ≡

```
⟨ RC_CovarianceInfluence Prototype 87b ⟩
{
    if (VARONLY) {
        RC_colSums(REAL(y), N, Q, Power2, ExpInf, DoCenter, weights,
                   subset, offset, Nsubset, PQ_ans);
        for (int q = 0; q < Q; q++)
            PQ_ans[q] = PQ_ans[q] / sumweights;
    } else {
        RC_KronSums(y, N, Q, REAL(y), Q, DoSymmetric, ExpInf, ExpInf, DoCenter, weights,
                    subset, offset, Nsubset, PQ_ans);
        for (int q = 0; q < Q * (Q + 1) / 2; q++)
            PQ_ans[q] = PQ_ans[q] / sumweights;
    }
}
◇
```

Fragment referenced in 82a.
Defines: `RC_CovarianceInfluence` 37b, 47, 87ab.
Uses: `DoCenter` 22b, `DoSymmetric` 22b, `N` 24bc, `Nsubset` 27c, `offset` 27d, `Power2` 22b, `Q` 25e, `RC_colSums` 114a, `RC_KronSums` 101a, `subset` 27be, 28a, `sumweights` 27a, `weights` 26c, `weights`, 26de, `y` 25d, 26ab.

3.8.3 X

⟨ *R_ExpectationX Prototype* 88b ⟩ ≡

```
SEXP R_ExpectationX
(
    ⟨ R x Input 24d ⟩
    SEXP P,
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩
)
◇
```

Fragment referenced in 23b, 89a.
Uses: `P` 25a, `R_ExpectationX` 89a.

⟨ *R_ExpectationX* 89a ⟩ ≡

```
⟨ R_ExpectationX Prototype 88b ⟩
{
    SEXP ans;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;

    N = XLENGTH(x) / INTEGER(P)[0];
    Nsubset = XLENGTH(subset);

    PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0]));
    RC_ExpectationX(x, N, INTEGER(P)[0], weights, subset,
                    Offset0, Nsubset, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 82a.
Defines: `R_ExpectationX` 88b, 92a, 164, 165.
Uses: `N` 24bc, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `RC_ExpectationX` 90, `subset` 27be, 28a, `weights` 26c, `weights`, 26de, `x` 24d, 25bc.

⟨ *RC_ExpectationX Prototype* 89b ⟩ ≡

```
void RC_ExpectationX
(
    ⟨ R x Input 24d ⟩
    ⟨ C integer N Input 24c ⟩,
    ⟨ C integer P Input 25a ⟩,
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    ⟨ C OneTableSums Answer 119c ⟩
)
◇
```

Fragment referenced in 90.
Uses: `RC_ExpectationX` 90.

⟨ *RC_ExpectationX* 90 ⟩ ≡

```
⟨ RC_ExpectationX Prototype 89b ⟩
{
    double center;

    if (TYPEOF(x) == INTSXP) {
        double* Pp1tmp = Calloc(P + 1, double);
        RC_OneTableSums(INTEGER(x), N, P + 1, weights, subset, offset, Nsubset, Pp1tmp);
        for (int p = 0; p < P; p++) P_ans[p] = Pp1tmp[p + 1];
        Free(Pp1tmp);
    } else {
        RC_colSums(REAL(x), N, P, Power1, &center, !DoCenter, weights, subset, offset, Nsubset, P_ans);
    }
}
◇
```

Fragment referenced in 82a.
Defines: `RC_ExpectationX` 37a, 46c, 89ab.
Uses: `DoCenter` 22b, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `Power1` 22b, `RC_colSums` 114a, `RC_OneTableSums` 119a, `subset` 27be, 28a, `weights` 26c, `weights`, 26de, `x` 24d, 25bc.

```
> a0 <- colSums(x[subset, ] * weights[subset])
> a0

[1] 59.67771 31.68129 47.29375

> a1 <- .Call(libcoin:::R_ExpectationX, x, P, weights, subset);
> a2 <- .Call(libcoin:::R_ExpectationX, x, P, as.double(weights), as.double(subset));
> a3 <- .Call(libcoin:::R_ExpectationX, x, P, weights, as.double(subset));
> a4 <- .Call(libcoin:::R_ExpectationX, x, P, as.double(weights), subset);
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4) &&
+           isequal(a0, LECVxyws$ExpectationX))
> a0 <- colSums(x[subset, ]^2 * weights[subset])
> a1 <- .Call(libcoin:::R_CovarianceX, x, P, weights, subset, 1L);
> a2 <- .Call(libcoin:::R_CovarianceX, x, P, as.double(weights), as.double(subset), 1L);
> a3 <- .Call(libcoin:::R_CovarianceX, x, P, weights, as.double(subset), 1L);
> a4 <- .Call(libcoin:::R_CovarianceX, x, P, as.double(weights), subset, 1L);
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
> a0 <- as.vector(colSums(Xfactor[subset, ] * weights[subset]))
> a0

 [1]  0 15  1  4  9  2 20  6  0 15

> a1 <- .Call(libcoin:::R_ExpectationX, ix, Lx, weights, subset);
> a2 <- .Call(libcoin:::R_ExpectationX, ix, Lx, as.double(weights), as.double(subset));
> a3 <- .Call(libcoin:::R_ExpectationX, ix, Lx, weights, as.double(subset));
> a4 <- .Call(libcoin:::R_ExpectationX, ix, Lx, as.double(weights), subset);
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
> a1 <- .Call(libcoin:::R_CovarianceX, ix, Lx, weights, subset, 1L);
> a2 <- .Call(libcoin:::R_CovarianceX, ix, Lx, as.double(weights), as.double(subset), 1L);
> a3 <- .Call(libcoin:::R_CovarianceX, ix, Lx, weights, as.double(subset), 1L);
> a4 <- .Call(libcoin:::R_CovarianceX, ix, Lx, as.double(weights), subset, 1L);
```

```
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
> r1x <- rep(1:ncol(Xfactor), ncol(Xfactor))
> r2x <- rep(1:ncol(Xfactor), each = ncol(Xfactor))
> a0 <- colSums(Xfactor[subset, r1x] * Xfactor[subset, r2x] * weights[subset])
> a0 <- matrix(a0, ncol = ncol(Xfactor))
> vary <- diag(a0)
> a0 <- a0[lower.tri(a0, diag = TRUE)]
> a1 <- .Call(libcoin:::R_CovarianceX, ix, Lx, weights, subset, 0L)
> a2 <- .Call(libcoin:::R_CovarianceX, ix, Lx, as.double(weights), as.double(subset), 0L)
> a3 <- .Call(libcoin:::R_CovarianceX, ix, Lx, weights, as.double(subset), 0L)
> a4 <- .Call(libcoin:::R_CovarianceX, ix, Lx, as.double(weights), subset, 0L)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

⟨ *R_CovarianceX Prototype* 91 ⟩ ≡

```
SEXP R_CovarianceX
(
    ⟨ R x Input 24d ⟩
    SEXP P,
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    SEXP varonly
)
◇
```

Fragment referenced in 23b, 92a.
Uses: `P` 25a, `R_CovarianceX` 92a.

⟨ *R_CovarianceX* 92a ⟩ ≡

```
⟨ R_CovarianceX Prototype 91 ⟩
{
    SEXP ans;
    SEXP ExpX;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;

    N = XLENGTH(x) / INTEGER(P)[0];
    Nsubset = XLENGTH(subset);

    PROTECT(ExpX = R_ExpectationX(x, P, weights, subset));

    if (INTEGER(varonly)[0]) {
        PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0]));
    } else {
        PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0] * (INTEGER(P)[0] + 1) / 2));
    }
    RC_CovarianceX(x, N, INTEGER(P)[0], weights, subset, Offset0, Nsubset, REAL(ExpX),
                   INTEGER(varonly)[0], REAL(ans));
    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 82a.
Defines: `R_CovarianceX` 91, 164, 165.
Uses: `N` 24bc, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `RC_CovarianceX` 93a, `R_ExpectationX` 89a, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc.

⟨ *RC_CovarianceX Prototype* 92b ⟩ ≡

```
void RC_CovarianceX
(
    ⟨ R x Input 24d ⟩
    ⟨ C integer N Input 24c ⟩,
    ⟨ C integer P Input 25a ⟩,
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    double *ExpX,
    int VARONLY,
    ⟨ C KronSums Answer 101d ⟩
)
◇
```

Fragment referenced in 93a.
Uses: `RC_CovarianceX` 93a.

⟨ *RC_CovarianceX* 93a ⟩ ≡

```
⟨ RC_CovarianceX Prototype 92b ⟩
{
    double center;

    if (TYPEOF(x) == INTSXP) {
        if (VARONLY) {
            for (int p = 0; p < P; p++) PQ_ans[p] = ExpX[p];
        } else {
            for (int p = 0; p < PP12(P); p++)
                PQ_ans[p] = 0.0;
            for (int p = 0; p < P; p++)
                PQ_ans[S(p, p, P)] = ExpX[p];
        }
    } else {
        if (VARONLY) {
            RC_colSums(REAL(x), N, P, Power2, &center, !DoCenter, weights,
                       subset, offset, Nsubset, PQ_ans);
        } else {
            RC_KronSums(x, N, P, REAL(x), P, DoSymmetric, &center, &center, !DoCenter, weights,
                        subset, offset, Nsubset, PQ_ans);
        }
    }
}
◇
```

Fragment referenced in 82a.
Defines: `RC_CovarianceX` 37c, 38a, 47, 92ab.
Uses: `DoCenter` 22b, `DoSymmetric` 22b, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `Power2` 22b, `PP12` 140b, `RC_colSums` 114a, `RC_KronSums` 101a, `S` 22a, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc.

3.9 Computing Sums

The core concept of all functions in the section is the computation of various sums over observations, weights, or blocks. We start with an initialisation of the loop over all observations

⟨ *init subset loop* 93b ⟩ ≡

```
R_xlen_t diff = 0;
s = subset + offset;
w = weights;
/* subset is R-style index in 1:N */
if (Nsubset > 0)
    diff = (R_xlen_t) s[0] - 1;
◇
```

Fragment referenced in 98a, 105, 108, 116b, 121b, 126, 131a.
Uses: `N` 24bc, `Nsubset` 27c, `offset` 27d, `subset` 27be, 28a, `weights` 26c.

and loop over $i = 1, \dots, N$ when no subset was specified or over the subset of the subset given by `offset` and `Nsubset`, allowing for number of observations larger than `INT_MAX`

⟨ *start subset loop* 94a ⟩ ≡

```
for (R_xlen_t i = 0; i < (Nsubset == 0 ? N : Nsubset) - 1; i++)
◇
```

Fragment referenced in 98a, 105, 108, 116b, 121b, 126, 131a.
Uses: `N` 24bc, `Nsubset` 27c.

After computions in the loop, we compute the next element

⟨ *continue subset loop* 94b ⟩ ≡

```
if (Nsubset > 0) {
    /* NB: diff also works with R style index */
    diff = (R_xlen_t) s[1] - s[0];
    if (diff < 0)
        error("subset not sorted");
    s++;
} else {
    diff = 1;
}
◇
```

Fragment referenced in 98a, 105, 108, 116b, 121b, 126, 131a.
Uses: `Nsubset` 27c, `subset` 27be, 28a.

3.9.1 Simple Sums

⟨ *SimpleSums* 94c ⟩ ≡

⟨ *C_Sums_dweights_dsubset* 96b ⟩
⟨ *C_Sums_iweights_dsubset* 97a ⟩
⟨ *C_Sums_iweights_isubset* 97b ⟩
⟨ *C_Sums_dweights_isubset* 97c ⟩
⟨ *RC_Sums* 96a ⟩
⟨ *R_Sums* 95b ⟩
◇

Fragment referenced in 24a.

```
> a0 <- sum(weights[subset])
> a1 <- .Call(libcoin:::R_Sums, N, weights, subset)
> a2 <- .Call(libcoin:::R_Sums, N, as.double(weights), as.double(subset))
> a3 <- .Call(libcoin:::R_Sums, N, weights, as.double(subset))
> a4 <- .Call(libcoin:::R_Sums, N, as.double(weights), subset)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

⟨ *R_Sums Prototype* 95a ⟩ ≡

```
SEXP R_Sums
(
    ⟨ R N Input 24b ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩
)
◇
```

Fragment referenced in 23b, 95b.
Uses: `R_Sums` 95b.

⟨ *R_Sums* 95b ⟩ ≡

```
⟨ R_Sums Prototype 95a ⟩
{
    SEXP ans;
    ⟨ C integer Nsubset Input 27c ⟩;

    Nsubset = XLENGTH(subset);

    PROTECT(ans = allocVector(REALSXP, 1));
    REAL(ans)[0] = RC_Sums(INTEGER(N)[0], weights, subset, Offset0, Nsubset);
    UNPROTECT(1);

    return(ans);
}
◇
```

Fragment referenced in 94c.
Defines: `R_Sums` 95a, 164, 165.
Uses: `N` 24bc, `Nsubset` 27c, `Offset0` 22b, `RC_Sums` 96a, `subset` 27be, 28a, `weights` 26c, `weights,` 26de.

⟨ *RC_Sums Prototype* 95c ⟩ ≡

```
double RC_Sums
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩
)
◇
```

Fragment referenced in 96a.
Uses: `RC_Sums` 96a.

⟨ *RC_Sums* 96a ⟩ ≡

```
⟨ RC_Sums Prototype 95c ⟩
{
    if (XLENGTH(weights) == 0) {
        if (XLENGTH(subset) == 0) {
            return((double) N);
        } else {
            return((double) Nsubset);
        }
    }
    if (TYPEOF(weights) == INTSXP) {
        if (TYPEOF(subset) == INTSXP) {
            return(C_Sums_iweights_isubset(N, INTEGER(weights), XLENGTH(weights),
                                           INTEGER(subset), offset, Nsubset));
        } else {
            return(C_Sums_iweights_dsubset(N, INTEGER(weights), XLENGTH(weights),
                                           REAL(subset), offset, Nsubset));
        }
    } else {
        if (TYPEOF(subset) == INTSXP) {
            return(C_Sums_dweights_isubset(N, REAL(weights), XLENGTH(weights),
                                           INTEGER(subset), offset, Nsubset));
        } else {
            return(C_Sums_dweights_dsubset(N, REAL(weights), XLENGTH(weights),
                                           REAL(subset), offset, Nsubset));
        }
    }
}
◇
```

Fragment referenced in 94c.
Defines: `RC_Sums` 36ab, 85b, 87a, 95bc, 132b, 136a.
Uses: `C_Sums_dweights_dsubset` 96b, `C_Sums_dweights_isubset` 97c, `C_Sums_iweights_dsubset` 97a, `C_Sums_iweights_isubset` 97b, `N` 24bc, `Nsubset` 27c, `offset` 27d, `subset` 27be, 28a, `weights` 26c.

⟨ *C_Sums_dweights_dsubset* 96b ⟩ ≡

```
double C_Sums_dweights_dsubset
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ C real weights Input 26e ⟩
    ⟨ C real subset Input 28a ⟩
) {
    double *s, *w;
    ⟨ Sums Body 98a ⟩
}
◇
```

Fragment referenced in 94c.
Defines: `C_Sums_dweights_dsubset` 96a.

⟨ *C_Sums_iweights_dsubset* 97a ⟩ ≡

```
double C_Sums_iweights_dsubset
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ C integer weights Input 26d ⟩
    ⟨ C real subset Input 28a ⟩
) {
    double *s;
    int *w;
    ⟨ Sums Body 98a ⟩
}
◇
```

Fragment referenced in 94c.
Defines: `C_Sums_iweights_dsubset` 96a.

⟨ *C_Sums_iweights_isubset* 97b ⟩ ≡

```
double C_Sums_iweights_isubset
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ C integer weights Input 26d ⟩
    ⟨ C integer subset Input 27e ⟩
) {
    int *s, *w;
    ⟨ Sums Body 98a ⟩
}
◇
```

Fragment referenced in 94c.
Defines: `C_Sums_iweights_isubset` 96a.

⟨ *C_Sums_dweights_isubset* 97c ⟩ ≡

```
double C_Sums_dweights_isubset
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ C real weights Input 26e ⟩
    ⟨ C integer subset Input 27e ⟩
) {
    int *s;
    double *w;
    ⟨ Sums Body 98a ⟩
}
◇
```

Fragment referenced in 94c.
Defines: `C_Sums_dweights_isubset` 96a.

⟨ *Sums Body* 98a ⟩ ≡

```
double ans = 0.0;

if (Nsubset > 0) {
    if (!HAS_WEIGHTS) return((double) Nsubset);
} else {
    if (!HAS_WEIGHTS) return((double) N);
}

⟨ init subset loop 93b ⟩
⟨ start subset loop 94a ⟩
{
    w = w + diff;
    ans += w[0];
    ⟨ continue subset loop 94b ⟩
}
w = w + diff;
ans += w[0];

return(ans);
◇
```

Fragment referenced in 96b, 97abc.
Uses: `HAS_WEIGHTS` 26de, `N` 24bc, `Nsubset` 27c.

3.9.2 Kronecker Sums

⟨ *KronSums* 98b ⟩ ≡

```
⟨ C_KronSums_dweights_dsubset 103b ⟩
⟨ C_KronSums_iweights_dsubset 104a ⟩
⟨ C_KronSums_iweights_isubset 104b ⟩
⟨ C_KronSums_dweights_isubset 104c ⟩
⟨ C_XfactorKronSums_dweights_dsubset 106b ⟩
⟨ C_XfactorKronSums_iweights_dsubset 106c ⟩
⟨ C_XfactorKronSums_iweights_isubset 107a ⟩
⟨ C_XfactorKronSums_dweights_isubset 107b ⟩
⟨ RC_KronSums 101a ⟩
⟨ R_KronSums 100a ⟩
⟨ C_KronSums_Permutation_isubset 111a ⟩
⟨ C_KronSums_Permutation_dsubset 110b ⟩
⟨ C_XfactorKronSums_Permutation_isubset 112a ⟩
⟨ C_XfactorKronSums_Permutation_dsubset 111c ⟩
⟨ RC_KronSums_Permutation 110a ⟩
⟨ R_KronSums_Permutation 109b ⟩
◇
```

Fragment referenced in 24a.

```
> r1 <- rep(1:ncol(x), ncol(y))
> r2 <- rep(1:ncol(y), each = ncol(x))
> a0 <- colSums(x[subset,r1] * y[subset,r2] * weights[subset])
> a1 <- .Call(libcoin:::R_KronSums, x, P, y, weights, subset, 0L)
> a2 <- .Call(libcoin:::R_KronSums, x, P, y, as.double(weights), as.double(subset), 0L)
> a3 <- .Call(libcoin:::R_KronSums, x, P, y, weights, as.double(subset), 0L)
```

```
> a4 <- .Call(libcoin:::R_KronSums, x, P, y, as.double(weights), subset, 0L)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
> a0 <- as.vector(colSums(Xfactor[subset,r1Xfactor] *
+                         y[subset,r2Xfactor] * weights[subset]))
> a1 <- .Call(libcoin:::R_KronSums, ix, Lx, y, weights, subset, 0L)
> a2 <- .Call(libcoin:::R_KronSums, ix, Lx, y, as.double(weights), as.double(subset), 0L)
> a3 <- .Call(libcoin:::R_KronSums, ix, Lx, y, weights, as.double(subset), 0L)
> a4 <- .Call(libcoin:::R_KronSums, ix, Lx, y, as.double(weights), subset, 0L)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

⟨ *R_KronSums Prototype* 99 ⟩ ≡

```
SEXP R_KronSums
(
    ⟨ R x Input 24d ⟩
    SEXP P,
    ⟨ R y Input 25d ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    SEXP symmetric
)
◇
```

Fragment referenced in 23b, 100a.
Uses: `P` 25a, `R_KronSums` 100a.

⟨ *R_KronSums* 100a ⟩ ≡

```
⟨ R_KronSums Prototype 99 ⟩
{
    SEXP ans;
    ⟨ C integer Q Input 25e ⟩;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;

    double center;

    Q = NCOL(y);
    N = XLENGTH(y) / Q;
    Nsubset = XLENGTH(subset);

    if (INTEGER(symmetric)[0]) {
        PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0] * (INTEGER(P)[0] + 1) / 2));
    } else {
        PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0] * Q));
    }
    RC_KronSums(x, N, INTEGER(P)[0], REAL(y), Q, INTEGER(symmetric)[0], &center, &center,
                !DoCenter, weights, subset, Offset0, Nsubset, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 98b.
Defines: `R_KronSums` 99, 164, 165.
Uses: `DoCenter` 22b, `N` 24bc, `NCOL` 139c, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `Q` 25e, `RC_KronSums` 101a, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *RC_KronSums Prototype* 100b ⟩ ≡

```
void RC_KronSums
(
    ⟨ RC KronSums Input 101b ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    ⟨ C KronSums Answer 101d ⟩
)
◇
```

Fragment referenced in 101a.
Uses: `RC_KronSums` 101a.

⟨ *RC_KronSums* 101a ⟩ ≡

```
⟨ RC_KronSums Prototype 100b ⟩
{
    if (TYPEOF(x) == INTSXP) {
        ⟨ KronSums Integer x 102 ⟩
    } else {
        ⟨ KronSums Double x 103a ⟩
    }
}
◇
```

Fragment referenced in 98b.
Defines: `RC_KronSums` 81d, 88a, 93a, 100ab.
Uses: `x` 24d, 25bc.

⟨ *RC KronSums Input* 101b ⟩ ≡

```
⟨ R x Input 24d ⟩
⟨ C integer N Input 24c ⟩,
⟨ C integer P Input 25a ⟩,
⟨ C real y Input 26a ⟩
const int SYMMETRIC,
double *centerx,
double *centery,
const int CENTER,
◇
```

Fragment referenced in 100b.

⟨ *C KronSums Input* 101c ⟩ ≡

```
⟨ C real x Input 25b ⟩
⟨ C real y Input 26a ⟩
const int SYMMETRIC,
double *centerx,
double *centery,
const int CENTER,
◇
```

Fragment referenced in 103b, 104abc.

⟨ *C KronSums Answer* 101d ⟩ ≡

```
double *PQ_ans
◇
```

Fragment referenced in 81c, 87b, 92b, 100b, 103b, 104abc, 106bc, 107ab, 109c, 110b, 111ac, 112a.

⟨ *KronSums Integer x* 102 ⟩ ≡

```
if (SYMMETRIC) error("not implemented");
if (CENTER) error("not implemented");
if (TYPEOF(weights) == INTSXP) {
    if (TYPEOF(subset) == INTSXP) {
        C_XfactorKronSums_iweights_isubset(INTEGER(x), N, P, y, Q,
            INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
            offset, Nsubset, PQ_ans);
    } else {
        C_XfactorKronSums_iweights_dsubset(INTEGER(x), N, P, y, Q,
            INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
            offset, Nsubset, PQ_ans);
    }
} else {
    if (TYPEOF(subset) == INTSXP) {
        C_XfactorKronSums_dweights_isubset(INTEGER(x), N, P, y, Q,
            REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
            offset, Nsubset, PQ_ans);
    } else {
        C_XfactorKronSums_dweights_dsubset(INTEGER(x), N, P, y, Q,
            REAL(weights), XLENGTH(weights) > 0, REAL(subset),
            offset, Nsubset, PQ_ans);
    }
}
◇
```

Fragment referenced in 101a.
Uses: C_XfactorKronSums_dweights_dsubset 106b, C_XfactorKronSums_dweights_isubset 107b, C_XfactorKronSums_iweights_dsubset 106c, C_XfactorKronSums_iweights_isubset 107a, N 24bc, Nsubset 27c, offset 27d, P 25a, Q 25e, subset 27be, 28a, weights 26c, x 24d, 25bc, y 25d, 26ab.

⟨ *KronSums Double x* 103a ⟩ ≡

```
if (TYPEOF(weights) == INTSXP) {
    if (TYPEOF(subset) == INTSXP) {
        C_KronSums_iweights_isubset(REAL(x), N, P, y, Q, SYMMETRIC, centerx, centery, CENTER,
            INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
            offset, Nsubset, PQ_ans);
    } else {
        C_KronSums_iweights_dsubset(REAL(x), N, P, y, Q, SYMMETRIC, centerx, centery, CENTER,
            INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
            offset, Nsubset, PQ_ans);
    }
} else {
    if (TYPEOF(subset) == INTSXP) {
        C_KronSums_dweights_isubset(REAL(x), N, P, y, Q, SYMMETRIC, centerx, centery, CENTER,
            REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
            offset, Nsubset, PQ_ans);
    } else {
        C_KronSums_dweights_dsubset(REAL(x), N, P, y, Q, SYMMETRIC, centerx, centery, CENTER,
            REAL(weights), XLENGTH(weights) > 0, REAL(subset),
            offset, Nsubset, PQ_ans);
    }
}
◇
```

Fragment referenced in 101a.
Uses: `C_KronSums_dweights_dsubset` 103b, `C_KronSums_dweights_isubset` 104c, `C_KronSums_iweights_dsubset` 104a, `C_KronSums_iweights_isubset` 104b, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `Q` 25e, `subset` 27be, 28a, `weights` 26c, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *C_KronSums_dweights_dsubset* 103b ⟩ ≡

```
void C_KronSums_dweights_dsubset
(
    ⟨ C KronSums Input 101c ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C KronSums Answer 101d ⟩
) {
    double *s, *w;
    ⟨ KronSums Body 105 ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_KronSums_dweights_dsubset` 103a.

⟨ *C_KronSums_iweights_dsubset* 104a ⟩ ≡

```
void C_KronSums_iweights_dsubset
(
    ⟨ C KronSums Input 101c ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C KronSums Answer 101d ⟩
) {
    double *s;
    int *w;
    ⟨ KronSums Body 105 ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_KronSums_iweights_dsubset` 103a.

⟨ *C_KronSums_iweights_isubset* 104b ⟩ ≡

```
void C_KronSums_iweights_isubset
(
    ⟨ C KronSums Input 101c ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C KronSums Answer 101d ⟩
) {
    int *s, *w;
    ⟨ KronSums Body 105 ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_KronSums_iweights_isubset` 103a.

⟨ *C_KronSums_dweights_isubset* 104c ⟩ ≡

```
void C_KronSums_dweights_isubset
(
    ⟨ C KronSums Input 101c ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C KronSums Answer 101d ⟩
) {
    int *s;
    double *w;
    ⟨ KronSums Body 105 ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_KronSums_dweights_isubset` 103a.

⟨ *KronSums Body* 105 ⟩ ≡

```
double *xx, *yy, cx = 0.0, cy = 0.0, *thisPQ_ans;
int idx;

for (int p = 0; p < P; p++) {
    for (int q = (SYMMETRIC ? p : 0); q < Q; q++) {
        /* SYMMETRIC is column-wise, default
           is row-wise (maybe need to change this) */
        if (SYMMETRIC) {
            idx = S(p, q, P);
        } else {
            idx = q * P + p;
        }
        PQ_ans[idx] = 0.0;
        thisPQ_ans = PQ_ans + idx;
        yy = y + N * q;
        xx = x + N * p;

        if (CENTER) {
            cx = centerx[p];
            cy = centery[q];
        }
        ⟨ init subset loop 93b ⟩
        ⟨ start subset loop 94a ⟩
        {
            xx = xx + diff;
            yy = yy + diff;
            if (HAS_WEIGHTS) {
                w = w + diff;
                if (CENTER) {
                    thisPQ_ans[0] += (xx[0] - cx) * (yy[0] - cy) * w[0];
                } else {
                    thisPQ_ans[0] += xx[0] * yy[0] * w[0];
                }
            } else {
                if (CENTER) {
                    thisPQ_ans[0] += (xx[0] - cx) * (yy[0] - cy);
                } else {
                    thisPQ_ans[0] += xx[0] * yy[0];
                }
            }
            ⟨ continue subset loop 94b ⟩
        }
        xx = xx + diff;
        yy = yy + diff;
        if (HAS_WEIGHTS) {
            w = w + diff;
            thisPQ_ans[0] += (xx[0] - cx) * (yy[0] - cy) * w[0];
        } else {
            thisPQ_ans[0] += (xx[0] - cx) * (yy[0] - cy);
        }
    }
}
```
◇

Fragment referenced in 103b, 104abc.
Uses: HAS_WEIGHTS 26de, N 24bc, P 25a, Q 25e, S 22a, x 24d, 25bc, y 25d, 26ab.

Xfactor Kronecker Sums

⟨ *C XfactorKronSums Input* 106a ⟩ ≡

> ⟨ *C integer x Input* 25c ⟩
> ⟨ *C real y Input* 26a ⟩
> ◇

Fragment referenced in 106bc, 107ab.

⟨ *C_XfactorKronSums_dweights_dsubset* 106b ⟩ ≡

```
void C_XfactorKronSums_dweights_dsubset
(
    ⟨ C XfactorKronSums Input 106a ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C KronSums Answer 101d ⟩
) {
    double *s, *w;
    ⟨ XfactorKronSums Body 108 ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_XfactorKronSums_dweights_dsubset` 102.

⟨ *C_XfactorKronSums_iweights_dsubset* 106c ⟩ ≡

```
void C_XfactorKronSums_iweights_dsubset
(
    ⟨ C XfactorKronSums Input 106a ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C KronSums Answer 101d ⟩
) {
    double *s;
    int *w;
    ⟨ XfactorKronSums Body 108 ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_XfactorKronSums_iweights_dsubset` 102.

⟨ *C_XfactorKronSums_iweights_isubset* 107a ⟩ ≡

```
void C_XfactorKronSums_iweights_isubset
(
    ⟨ C XfactorKronSums Input 106a ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C KronSums Answer 101d ⟩
) {
    int *s, *w;
    ⟨ XfactorKronSums Body 108 ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_XfactorKronSums_iweights_isubset` 102.

⟨ *C_XfactorKronSums_dweights_isubset* 107b ⟩ ≡

```
void C_XfactorKronSums_dweights_isubset
(
    ⟨ C XfactorKronSums Input 106a ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C KronSums Answer 101d ⟩
) {
    int *s;
    double *w;
    ⟨ XfactorKronSums Body 108 ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_XfactorKronSums_dweights_isubset` 102.

⟨ *XfactorKronSums Body* 108 ⟩ ≡

```
int *xx, ixi;
double *yy;

for (int p = 0; p < mPQB(P, Q, 1); p++) PQ_ans[p] = 0.0;

for (int q = 0; q < Q; q++) {
    yy = y + N * q;
    xx = x;
    ⟨ init subset loop 93b ⟩
    ⟨ start subset loop 94a ⟩
    {
        xx = xx + diff;
        yy = yy + diff;
        ixi = xx[0] - 1;
        if (HAS_WEIGHTS) {
            w = w + diff;
            if (ixi >= 0)
                PQ_ans[ixi + q * P] += yy[0] * w[0];
        } else {
            if (ixi >= 0)
                PQ_ans[ixi + q * P] += yy[0];
        }
        ⟨ continue subset loop 94b ⟩
    }
    xx = xx + diff;
    yy = yy + diff;
    ixi = xx[0] - 1;
    if (HAS_WEIGHTS) {
        w = w + diff;
        if (ixi >= 0)
            PQ_ans[ixi + q * P] += yy[0] * w[0];
    } else {
        if (ixi >= 0)
            PQ_ans[ixi + q * P] += yy[0];
    }
}
◇
```

Fragment referenced in 106bc, 107ab.
Uses: `HAS_WEIGHTS` 26de, `mPQB` 141a, `N` 24bc, `P` 25a, `Q` 25e, `x` 24d, 25bc, `y` 25d, 26ab.

Permuted Kronecker Sums

```
> a0 <- colSums(x[subset,r1] * y[subsety, r2])
> a1 <- .Call(libcoin:::R_KronSums_Permutation, x, P, y, subset, subsety)
> a2 <- .Call(libcoin:::R_KronSums_Permutation, x, P, y, as.double(subset), as.double(subsety))
> stopifnot(isequal(a0, a1) && isequal(a0, a2))
> a0 <- as.vector(colSums(Xfactor[subset,r1Xfactor] * y[subsety, r2Xfactor]))
> a1 <- .Call(libcoin:::R_KronSums_Permutation, ix, Lx, y, subset, subsety)
> a2 <- .Call(libcoin:::R_KronSums_Permutation, ix, Lx, y, as.double(subset), as.double(subsety))
> stopifnot(isequal(a0, a1) && isequal(a0, a2))
```

⟨ *R_KronSums_Permutation Prototype* 109a ⟩ ≡

```
SEXP R_KronSums_Permutation
(
    ⟨ R x Input 24d ⟩
    SEXP P,
    ⟨ R y Input 25d ⟩
    ⟨ R subset Input 27b ⟩,
    SEXP subsety
)
◇
```

Fragment referenced in 23b, 109b.
Uses: `P` 25a, `R_KronSums_Permutation` 109b.

⟨ *R_KronSums_Permutation* 109b ⟩ ≡

```
⟨ R_KronSums_Permutation Prototype 109a ⟩
{
    SEXP ans;
    ⟨ C integer Q Input 25e ⟩;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;

    Q = NCOL(y);
    N = XLENGTH(y) / Q;
    Nsubset = XLENGTH(subset);

    PROTECT(ans = allocVector(REALSXP, INTEGER(P)[0] * Q));
    RC_KronSums_Permutation(x, N, INTEGER(P)[0], REAL(y), Q, subset, Offset0, Nsubset,
                            subsety, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 98b.
Defines: `R_KronSums_Permutation` 109a, 164, 165.
Uses: `N` 24bc, `NCOL` 139c, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `Q` 25e, `RC_KronSums_Permutation` 110a, `subset` 27be, 28a, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *RC_KronSums_Permutation Prototype* 109c ⟩ ≡

```
void RC_KronSums_Permutation
(
    ⟨ R x Input 24d ⟩
    ⟨ C integer N Input 24c ⟩,
    ⟨ C integer P Input 25a ⟩,
    ⟨ C real y Input 26a ⟩
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    SEXP subsety,
    ⟨ C KronSums Answer 101d ⟩
)
◇
```

Fragment referenced in 110a.
Uses: `RC_KronSums_Permutation` 110a.

⟨ *RC_KronSums_Permutation* 110a ⟩ ≡

```
⟨ RC_KronSums_Permutation Prototype 109c ⟩
{
    if (TYPEOF(x) == INTSXP) {
        if (TYPEOF(subset) == INTSXP) {
            C_XfactorKronSums_Permutation_isubset(INTEGER(x), N, P, y, Q,
                                                  INTEGER(subset), offset, Nsubset,
                                                  INTEGER(subsety), PQ_ans);
        } else {
            C_XfactorKronSums_Permutation_dsubset(INTEGER(x), N, P, y, Q,
                                                  REAL(subset), offset, Nsubset,
                                                  REAL(subsety), PQ_ans);
        }
    } else {
        if (TYPEOF(subset) == INTSXP) {
            C_KronSums_Permutation_isubset(REAL(x), N, P, y, Q,
                                           INTEGER(subset), offset, Nsubset,
                                           INTEGER(subsety), PQ_ans);
        } else {
            C_KronSums_Permutation_dsubset(REAL(x), N, P, y, Q,
                                           REAL(subset), offset, Nsubset,
                                           REAL(subsety), PQ_ans);
        }
    }
}
◇
```

Fragment referenced in 98b.
Defines: `RC_KronSums_Permutation` 40, 109bc.
Uses: `C_KronSums_Permutation_dsubset` 110b, `C_KronSums_Permutation_isubset` 111a, `C_XfactorKronSums_Permutation_dsubset` 111c, `C_XfactorKronSums_Permutation_isubset` 112a, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `Q` 25e, `subset` 27be, 28a, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *C_KronSums_Permutation_dsubset* 110b ⟩ ≡

```
void C_KronSums_Permutation_dsubset
(
    ⟨ C real x Input 25b ⟩
    ⟨ C real y Input 26a ⟩
    ⟨ C real subset Input 28a ⟩,
    double *subsety,
    ⟨ C KronSums Answer 101d ⟩
) {
    ⟨ KronSums Permutation Body 111b ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_KronSums_Permutation_dsubset` 110a.

⟨ *C_KronSums_Permutation_isubset* 111a ⟩ ≡

```
void C_KronSums_Permutation_isubset
(
    ⟨ C real x Input 25b ⟩
    ⟨ C real y Input 26a ⟩
    ⟨ C integer subset Input 27e ⟩,
    int *subsety,
    ⟨ C KronSums Answer 101d ⟩
) {
    ⟨ KronSums Permutation Body 111b ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_KronSums_Permutation_isubset` 110a.

Because `subset` might not be ordered (in the presence of blocks) we have to go through all elements explicitly here.

⟨ *KronSums Permutation Body* 111b ⟩ ≡

```
R_xlen_t qP, qN, pN, qPp;

for (int q = 0; q < Q; q++) {
    qN = q * N;
    qP = q * P;
    for (int p = 0; p < P; p++) {
        qPp = qP + p;
        PQ_ans[qPp] = 0.0;
        pN = p * N;
        for (R_xlen_t i = offset; i < Nsubset; i++)
            PQ_ans[qPp] += y[qN + (R_xlen_t) subsety[i] - 1] *
                           x[pN + (R_xlen_t) subset[i] - 1];
    }
}
◇
```

Fragment referenced in 110b, 111a.
Uses: `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `Q` 25e, `subset` 27be, 28a, `x` 24d, 25bc, `y` 25d, 26ab.

Xfactor Permuted Kronecker Sums

⟨ *C_XfactorKronSums_Permutation_dsubset* 111c ⟩ ≡

```
void C_XfactorKronSums_Permutation_dsubset
(
    ⟨ C integer x Input 25c ⟩
    ⟨ C real y Input 26a ⟩
    ⟨ C real subset Input 28a ⟩,
    double *subsety,
    ⟨ C KronSums Answer 101d ⟩
) {
    ⟨ XfactorKronSums Permutation Body 112b ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_XfactorKronSums_Permutation_dsubset` 110a.

⟨ *C_XfactorKronSums_Permutation_isubset* 112a ⟩ ≡

```
void C_XfactorKronSums_Permutation_isubset
(
    ⟨ C integer x Input 25c ⟩
    ⟨ C real y Input 26a ⟩
    ⟨ C integer subset Input 27e ⟩,
    int *subsety,
    ⟨ C KronSums Answer 101d ⟩
) {
    ⟨ XfactorKronSums Permutation Body 112b ⟩
}
◇
```

Fragment referenced in 98b.
Defines: `C_XfactorKronSums_Permutation_isubset` 110a.

⟨ *XfactorKronSums Permutation Body* 112b ⟩ ≡

```
R_xlen_t qP, qN;

for (int p = 0; p < mPQB(P, Q, 1); p++) PQ_ans[p] = 0.0;

for (int q = 0; q < Q; q++) {
    qP = q * P;
    qN = q * N;
    for (R_xlen_t i = offset; i < Nsubset; i++)
        PQ_ans[x[(R_xlen_t) subset[i] - 1] - 1 + qP] += y[qN + (R_xlen_t) subsety[i] - 1];
}
◇
```

Fragment referenced in 111c, 112a.
Uses: `mPQB` 141a, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `Q` 25e, `subset` 27be, 28a, `x` 24d, 25bc, `y` 25d, 26ab.

3.9.3 Column Sums

⟨ *colSums* 112c ⟩ ≡

```
⟨ C_colSums_dweights_dsubset 115a ⟩
⟨ C_colSums_iweights_dsubset 115b ⟩
⟨ C_colSums_iweights_isubset 115c ⟩
⟨ C_colSums_dweights_isubset 116a ⟩
⟨ RC_colSums 114a ⟩
⟨ R_colSums 113b ⟩
◇
```

Fragment referenced in 24a.

```
> a0 <- colSums(x[subset,] * weights[subset])
> a1 <- .Call(libcoin:::R_colSums, x, weights, subset)
> a2 <- .Call(libcoin:::R_colSums, x, as.double(weights), as.double(subset))
> a3 <- .Call(libcoin:::R_colSums, x, weights, as.double(subset))
> a4 <- .Call(libcoin:::R_colSums, x, as.double(weights), subset)
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

⟨ *R_colSums Prototype* 113a ⟩ ≡

```
SEXP R_colSums
(
    ⟨ R x Input 24d ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩
)
◇
```

Fragment referenced in 23b, 113b.
Uses: `R_colSums` 113b.

⟨ *R_colSums* 113b ⟩ ≡

```
⟨ R_colSums Prototype 113a ⟩
{
    SEXP ans;
    int P;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;
    double center;

    P = NCOL(x);
    N = XLENGTH(x) / P;
    Nsubset = XLENGTH(subset);

    PROTECT(ans = allocVector(REALSXP, P));
    RC_colSums(REAL(x), N, P, Power1, &center, !DoCenter, weights, subset, Offset0,
               Nsubset, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 112c.
Defines: `R_colSums` 113a, 164, 165.
Uses: `DoCenter` 22b, `N` 24bc, `NCOL` 139c, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `Power1` 22b, `RC_colSums` 114a, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc.

⟨ *RC_colSums Prototype* 113c ⟩ ≡

```
void RC_colSums
(
    ⟨ C colSums Input 114b ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    ⟨ C colSums Answer 114c ⟩
)
◇
```

Fragment referenced in 114a.
Uses: `RC_colSums` 114a.

⟨ *RC_colSums* 114a ⟩ ≡

```
⟨ RC_colSums Prototype 113c ⟩
{
    if (TYPEOF(weights) == INTSXP) {
        if (TYPEOF(subset) == INTSXP) {
            C_colSums_iweights_isubset(x, N, P, power, centerx, CENTER,
                                       INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                       offset, Nsubset, P_ans);
        } else {
            C_colSums_iweights_dsubset(x, N, P, power, centerx, CENTER,
                                       INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
                                       offset, Nsubset, P_ans);
        }
    } else {
        if (TYPEOF(subset) == INTSXP) {
            C_colSums_dweights_isubset(x, N, P, power, centerx, CENTER,
                                       REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                       offset, Nsubset, P_ans);
        } else {
            C_colSums_dweights_dsubset(x, N, P, power, centerx, CENTER,
                                       REAL(weights), XLENGTH(weights) > 0, REAL(subset),
                                       offset, Nsubset, P_ans);
        }
    }
}
◇
```

Fragment referenced in 112c.
Defines: `RC_colSums` 86a, 88a, 90, 93a, 113bc.
Uses: `C_colSums_dweights_dsubset` 115a, `C_colSums_dweights_isubset` 116a, `C_colSums_iweights_dsubset` 115b, `C_colSums_iweights_isubset` 115c, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `subset` 27be, 28a, `weights` 26c, `x` 24d, 25bc.

⟨ *C colSums Input* 114b ⟩ ≡

```
⟨ C real x Input 25b ⟩
const int power,
double *centerx,
const int CENTER,
◇
```

Fragment referenced in 113c, 115abc, 116a.

⟨ *C colSums Answer* 114c ⟩ ≡

```
double *P_ans
◇
```

Fragment referenced in 85c, 113c, 115abc, 116a.

⟨ *C_colSums_dweights_dsubset* 115a ⟩ ≡

```
void C_colSums_dweights_dsubset
(
    ⟨ C colSums Input 114b ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C colSums Answer 114c ⟩
) {
    double *s, *w;
    ⟨ colSums Body 116b ⟩
}
◇
```

Fragment referenced in 112c.
Defines: `C_colSums_dweights_dsubset` 114a.

⟨ *C_colSums_iweights_dsubset* 115b ⟩ ≡

```
void C_colSums_iweights_dsubset
(
    ⟨ C colSums Input 114b ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C colSums Answer 114c ⟩
) {
    double *s;
    int *w;
    ⟨ colSums Body 116b ⟩
}
◇
```

Fragment referenced in 112c.
Defines: `C_colSums_iweights_dsubset` 114a.

⟨ *C_colSums_iweights_isubset* 115c ⟩ ≡

```
void C_colSums_iweights_isubset
(
    ⟨ C colSums Input 114b ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C colSums Answer 114c ⟩
) {
    int *s, *w;
    ⟨ colSums Body 116b ⟩
}
◇
```

Fragment referenced in 112c.
Defines: `C_colSums_iweights_isubset` 114a.

⟨ *C_colSums_dweights_isubset* 116a ⟩ ≡

```
void C_colSums_dweights_isubset
(
    ⟨ C colSums Input 114b ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C colSums Answer 114c ⟩
) {
    int *s;
    double *w;
    ⟨ colSums Body 116b ⟩
}
◇
```

Fragment referenced in 112c.
Defines: `C_colSums_dweights_isubset` 114a.

⟨ *colSums Body* 116b ⟩ ≡

```
double *xx, cx = 0.0;

for (int p = 0; p < P; p++) {
    P_ans[0] = 0.0;
    xx = x + N * p;
    if (CENTER) {
        cx = centerx[p];
    }
    ⟨ init subset loop 93b ⟩
    ⟨ start subset loop 94a ⟩
    {
        xx = xx + diff;
        if (HAS_WEIGHTS) {
            w = w + diff;
            P_ans[0] += pow(xx[0] - cx, power) * w[0];
        } else {
            P_ans[0] += pow(xx[0] - cx, power);
        }
        ⟨ continue subset loop 94b ⟩
    }
    xx = xx + diff;
    if (HAS_WEIGHTS) {
        w = w + diff;
        P_ans[0] += pow(xx[0] - cx, power) * w[0];
    } else {
        P_ans[0] += pow(xx[0] - cx, power);
    }
    P_ans++;
}
◇
```

Fragment referenced in 115abc, 116a.
Uses: `HAS_WEIGHTS` 26de, `N` 24bc, `P` 25a, `x` 24d, 25bc.

3.9.4 Tables

OneTable Sums

⟨ *Tables* 117a ⟩ ≡

> ⟨ *C_OneTableSums_dweights_dsubset* 120a ⟩
> ⟨ *C_OneTableSums_iweights_dsubset* 120b ⟩
> ⟨ *C_OneTableSums_iweights_isubset* 120c ⟩
> ⟨ *C_OneTableSums_dweights_isubset* 121a ⟩
> ⟨ *RC_OneTableSums* 119a ⟩
> ⟨ *R_OneTableSums* 118a ⟩
> ⟨ *C_TwoTableSums_dweights_dsubset* 124b ⟩
> ⟨ *C_TwoTableSums_iweights_dsubset* 124c ⟩
> ⟨ *C_TwoTableSums_iweights_isubset* 125a ⟩
> ⟨ *C_TwoTableSums_dweights_isubset* 125b ⟩
> ⟨ *RC_TwoTableSums* 123b ⟩
> ⟨ *R_TwoTableSums* 122b ⟩
> ⟨ *C_ThreeTableSums_dweights_dsubset* 129b ⟩
> ⟨ *C_ThreeTableSums_iweights_dsubset* 129c ⟩
> ⟨ *C_ThreeTableSums_iweights_isubset* 130a ⟩
> ⟨ *C_ThreeTableSums_dweights_isubset* 130b ⟩
> ⟨ *RC_ThreeTableSums* 128b ⟩
> ⟨ *R_ThreeTableSums* 127b ⟩
> ◇

Fragment referenced in 24a.

```
> a0 <- as.vector(xtabs(weights ~ ixf, subset = subset))
> a1 <- ctabs(ix, weights = weights, subset = subset)[-1]
> a2 <- ctabs(ix, weights = as.double(weights), subset = as.double(subset))[-1]
> a3 <- ctabs(ix, weights = weights, subset = as.double(subset))[-1]
> a4 <- ctabs(ix, weights = as.double(weights), subset = subset)[-1]
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

⟨ *R_OneTableSums Prototype* 117b ⟩ ≡

```
SEXP R_OneTableSums
(
    ⟨ R x Input 24d ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩
)
◇
```

Fragment referenced in 23b, 118a.
Uses: `R_OneTableSums` 118a.

⟨ *R_OneTableSums* 118a ⟩ ≡

```
⟨ R_OneTableSums Prototype 117b ⟩
{
    SEXP ans;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;
    int P;

    N = XLENGTH(x);
    Nsubset = XLENGTH(subset);
    P = NLEVELS(x) + 1;

    PROTECT(ans = allocVector(REALSXP, P));
    RC_OneTableSums(INTEGER(x), N, P, weights, subset,
                    Offset0, Nsubset, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
◇
```

Fragment referenced in 117a.
Defines: `R_OneTableSums` 16, 117b, 132b, 164, 165.
Uses: `N` 24bc, `NLEVELS` 140a, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `RC_OneTableSums` 119a, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc.

⟨ *RC_OneTableSums Prototype* 118b ⟩ ≡

```
void RC_OneTableSums
(
    ⟨ C OneTableSums Input 119b ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    ⟨ C OneTableSums Answer 119c ⟩
)
◇
```

Fragment referenced in 119a.
Uses: `RC_OneTableSums` 119a.

⟨ *RC_OneTableSums* 119a ⟩ ≡

```
⟨ RC_OneTableSums Prototype 118b ⟩
{
    if (TYPEOF(weights) == INTSXP) {
        if (TYPEOF(subset) == INTSXP) {
            C_OneTableSums_iweights_isubset(x, N, P,
                                            INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                            offset, Nsubset, P_ans);
        } else {
            C_OneTableSums_iweights_dsubset(x, N, P,
                                            INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
                                            offset, Nsubset, P_ans);
        }
    } else {
        if (TYPEOF(subset) == INTSXP) {
            C_OneTableSums_dweights_isubset(x, N, P,
                                            REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                            offset, Nsubset, P_ans);
        } else {
            C_OneTableSums_dweights_dsubset(x, N, P,
                                            REAL(weights), XLENGTH(weights) > 0, REAL(subset),
                                            offset, Nsubset, P_ans);
        }
    }
}
◇
```

Fragment referenced in 117a.
Defines: `RC_OneTableSums` 36a, 40, 90, 118ab.
Uses: `C_OneTableSums_dweights_dsubset` 120a, `C_OneTableSums_dweights_isubset` 121a, `C_OneTableSums_iweights_dsubset` 120b, `C_OneTableSums_iweights_isubset` 120c, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `subset` 27be, 28a, `weights` 26c, `x` 24d, 25bc.

⟨ *C OneTableSums Input* 119b ⟩ ≡

```
⟨ C integer x Input 25c ⟩
◇
```

Fragment referenced in 118b, 120abc, 121a.

⟨ *C OneTableSums Answer* 119c ⟩ ≡

```
double *P_ans
◇
```

Fragment referenced in 89b, 118b, 120abc, 121a.

⟨ *C_OneTableSums_dweights_dsubset* 120a ⟩ ≡

```
void C_OneTableSums_dweights_dsubset
(
    ⟨ C OneTableSums Input 119b ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C OneTableSums Answer 119c ⟩
) {
    double *s, *w;
    ⟨ OneTableSums Body 121b ⟩
}
◇
```

Fragment referenced in 117a.
Defines: `C_OneTableSums_dweights_dsubset` 119a.

⟨ *C_OneTableSums_iweights_dsubset* 120b ⟩ ≡

```
void C_OneTableSums_iweights_dsubset
(
    ⟨ C OneTableSums Input 119b ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C OneTableSums Answer 119c ⟩
) {
    double *s;
    int *w;
    ⟨ OneTableSums Body 121b ⟩
}
◇
```

Fragment referenced in 117a.
Defines: `C_OneTableSums_iweights_dsubset` 119a.

⟨ *C_OneTableSums_iweights_isubset* 120c ⟩ ≡

```
void C_OneTableSums_iweights_isubset
(
    ⟨ C OneTableSums Input 119b ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C OneTableSums Answer 119c ⟩
) {
    int *s, *w;
    ⟨ OneTableSums Body 121b ⟩
}
◇
```

Fragment referenced in 117a.
Defines: `C_OneTableSums_iweights_isubset` 119a.

⟨ *C_OneTableSums_dweights_isubset* 121a ⟩ ≡

```
void C_OneTableSums_dweights_isubset
(
    ⟨ C OneTableSums Input 119b ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C OneTableSums Answer 119c ⟩
) {
    int *s;
    double *w;
    ⟨ OneTableSums Body 121b ⟩
}
◇
```

Fragment referenced in 117a.
Defines: `C_OneTableSums_dweights_isubset` 119a.

⟨ *OneTableSums Body* 121b ⟩ ≡

```
int *xx;

for (int p = 0; p < P; p++) P_ans[p] = 0.0;

xx = x;
⟨ init subset loop 93b ⟩
⟨ start subset loop 94a ⟩
{
    xx = xx + diff;
    if (HAS_WEIGHTS) {
        w = w + diff;
        P_ans[xx[0]] += (double) w[0];
    } else {
        P_ans[xx[0]]++;
    }
    ⟨ continue subset loop 94b ⟩
}
xx = xx + diff;
if (HAS_WEIGHTS) {
    w = w + diff;
    P_ans[xx[0]] += w[0];
} else {
    P_ans[xx[0]]++;
}
◇
```

Fragment referenced in 120abc, 121a.
Uses: `HAS_WEIGHTS` 26de, `P` 25a, `x` 24d, 25bc.

TwoTable Sums

```
> a0 <- xtabs(weights ~ ixf + iyf, subset = subset)
> class(a0) <- "matrix"
> dimnames(a0) <- NULL
> attributes(a0)$call <- NULL
> a1 <- ctabs(ix, iy, weights = weights, subset = subset)[-1, -1]
```

```
> a2 <- ctabs(ix, iy, weights = as.double(weights),
+               subset = as.double(subset))[-1, -1]
> a3 <- ctabs(ix, iy, weights = weights, subset = as.double(subset))[-1, -1]
> a4 <- ctabs(ix, iy, weights = as.double(weights), subset = subset)[-1, -1]
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+             isequal(a0, a3) && isequal(a0, a4))
```

⟨ *R_TwoTableSums Prototype* 122a ⟩ ≡

```
SEXP R_TwoTableSums
(
    ⟨ R x Input 24d ⟩
    ⟨ R y Input 25d ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩
)
◇
```

Fragment referenced in 23b, 122b.
Uses: `R_TwoTableSums` 122b.

⟨ *R_TwoTableSums* 122b ⟩ ≡

```
⟨ R_TwoTableSums Prototype 122a ⟩
{
    SEXP ans, dim;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;
    int P, Q;

    N = XLENGTH(x);
    Nsubset = XLENGTH(subset);
    P = NLEVELS(x) + 1;
    Q = NLEVELS(y) + 1;

    PROTECT(ans = allocVector(REALSXP, mPQB(P, Q, 1)));
    PROTECT(dim = allocVector(INTSXP, 2));
    INTEGER(dim)[0] = P;
    INTEGER(dim)[1] = Q;
    dimgets(ans, dim);
    RC_TwoTableSums(INTEGER(x), N, P, INTEGER(y), Q,
                    weights, subset, Offset0, Nsubset, REAL(ans));
    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 117a.
Defines: `R_TwoTableSums` 16, 122a, 164, 165.
Uses: `mPQB` 141a, `N` 24bc, `NLEVELS` 140a, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `Q` 25e, `RC_TwoTableSums` 123b, `subset` 27be, 28a, `weights` 26c, `weights,` 26de, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *RC_TwoTableSums Prototype* 123a ⟩ ≡

```
void RC_TwoTableSums
(
    ⟨ C TwoTableSums Input 123c ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    ⟨ C TwoTableSums Answer 124a ⟩
)
◇
```

Fragment referenced in 123b.
Uses: `RC_TwoTableSums` 123b.

⟨ *RC_TwoTableSums* 123b ⟩ ≡

```
⟨ RC_TwoTableSums Prototype 123a ⟩
{
    if (TYPEOF(weights) == INTSXP) {
        if (TYPEOF(subset) == INTSXP) {
            C_TwoTableSums_iweights_isubset(x, N, P, y, Q,
                                            INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                            offset, Nsubset, PQ_ans);
        } else {
            C_TwoTableSums_iweights_dsubset(x, N, P, y, Q,
                                            INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
                                            offset, Nsubset, PQ_ans);
        }
    } else {
        if (TYPEOF(subset) == INTSXP) {
            C_TwoTableSums_dweights_isubset(x, N, P, y, Q,
                                            REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                            offset, Nsubset, PQ_ans);
        } else {
            C_TwoTableSums_dweights_dsubset(x, N, P, y, Q,
                                            REAL(weights), XLENGTH(weights) > 0, REAL(subset),
                                            offset, Nsubset, PQ_ans);
        }
    }
}
◇
```

Fragment referenced in 117a.
Defines: `RC_TwoTableSums` 44, 122b, 123a.
Uses: `C_TwoTableSums_dweights_dsubset` 124b, `C_TwoTableSums_dweights_isubset` 125b, `C_TwoTableSums_iweights_dsubset` 124c, `C_TwoTableSums_iweights_isubset` 125a, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `Q` 25e, `subset` 27be, 28a, `weights` 26c, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *C TwoTableSums Input* 123c ⟩ ≡

```
⟨ C integer x Input 25c ⟩
⟨ C integer y Input 26b ⟩
◇
```

Fragment referenced in 123a, 124bc, 125ab.

⟨ *C TwoTableSums Answer* 124a ⟩ ≡

```
double *PQ_ans
```
◇

Fragment referenced in 123a, 124bc, 125ab.

⟨ *C_TwoTableSums_dweights_dsubset* 124b ⟩ ≡

```
void C_TwoTableSums_dweights_dsubset
(
    ⟨ C TwoTableSums Input 123c ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C TwoTableSums Answer 124a ⟩
) {
    double *s, *w;
    ⟨ TwoTableSums Body 126 ⟩
}
```
◇

Fragment referenced in 117a.
Defines: `C_TwoTableSums_dweights_dsubset` 123b.

⟨ *C_TwoTableSums_iweights_dsubset* 124c ⟩ ≡

```
void C_TwoTableSums_iweights_dsubset
(
    ⟨ C TwoTableSums Input 123c ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C TwoTableSums Answer 124a ⟩
) {
    double *s;
    int *w;
    ⟨ TwoTableSums Body 126 ⟩
}
```
◇

Fragment referenced in 117a.
Defines: `C_TwoTableSums_iweights_dsubset` 123b.

⟨ *C_TwoTableSums_iweights_isubset* 125a ⟩ ≡

```
void C_TwoTableSums_iweights_isubset
(
    ⟨ C TwoTableSums Input 123c ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C TwoTableSums Answer 124a ⟩
) {
    int *s, *w;
    ⟨ TwoTableSums Body 126 ⟩
}
◇
```

Fragment referenced in 117a.
Defines: `C_TwoTableSums_iweights_isubset` 123b.

⟨ *C_TwoTableSums_dweights_isubset* 125b ⟩ ≡

```
void C_TwoTableSums_dweights_isubset
(
    ⟨ C TwoTableSums Input 123c ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C TwoTableSums Answer 124a ⟩
) {
    int *s;
    double *w;
    ⟨ TwoTableSums Body 126 ⟩
}
◇
```

Fragment referenced in 117a.
Defines: `C_TwoTableSums_dweights_isubset` 123b.

⟨ *TwoTableSums Body* 126 ⟩ ≡

```
int *xx, *yy;

for (int p = 0; p < Q * P; p++) PQ_ans[p] = 0.0;

yy = y;
xx = x;
⟨ init subset loop 93b ⟩
⟨ start subset loop 94a ⟩
{
    xx = xx + diff;
    yy = yy + diff;
    if (HAS_WEIGHTS) {
        w = w + diff;
        PQ_ans[yy[0] * P + xx[0]] += (double) w[0];
    } else {
        PQ_ans[yy[0] * P + xx[0]]++;
    }
    ⟨ continue subset loop 94b ⟩
}
xx = xx + diff;
yy = yy + diff;
if (HAS_WEIGHTS) {
    w = w + diff;
    PQ_ans[yy[0] * P + xx[0]] += w[0];
} else {
    PQ_ans[yy[0] * P + xx[0]]++;
}
◇
```

Fragment referenced in 124bc, 125ab.
Uses: `HAS_WEIGHTS` 26de, `P` 25a, `Q` 25e, `x` 24d, 25bc, `y` 25d, 26ab.

ThreeTable Sums

```
> a0 <- xtabs(weights ~ ixf + iyf + block, subset = subset)
> class(a0) <- "array"
> dimnames(a0) <- NULL
> attributes(a0)$call <- NULL
> a1 <- ctabs(ix, iy, block, weights, subset)[-1, -1,]
> a2 <- ctabs(ix, iy, block, as.double(weights), as.double(subset))[-1,-1,]
> a3 <- ctabs(ix, iy, block, weights, as.double(subset))[-1,-1,]
> a4 <- ctabs(ix, iy, block, as.double(weights), subset)[-1,-1,]
> stopifnot(isequal(a0, a1) && isequal(a0, a2) &&
+           isequal(a0, a3) && isequal(a0, a4))
```

⟨ *R_ThreeTableSums Prototype* 127a ⟩ ≡

```
SEXP R_ThreeTableSums
(
    ⟨ R x Input 24d ⟩
    ⟨ R y Input 25d ⟩
    ⟨ R block Input 28b ⟩,
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩
)
◇
```

Fragment referenced in 23b, 127b.
Uses: `R_ThreeTableSums` 127b.

⟨ *R_ThreeTableSums* 127b ⟩ ≡

```
⟨ R_ThreeTableSums Prototype 127a ⟩
{
    SEXP ans, dim;
    ⟨ C integer N Input 24c ⟩;
    ⟨ C integer Nsubset Input 27c ⟩;
    int P, Q, B;

    N = XLENGTH(x);
    Nsubset = XLENGTH(subset);
    P = NLEVELS(x) + 1;
    Q = NLEVELS(y) + 1;
    B = NLEVELS(block);

    PROTECT(ans = allocVector(REALSXP, mPQB(P, Q, B)));
    PROTECT(dim = allocVector(INTSXP, 3));
    INTEGER(dim)[0] = P;
    INTEGER(dim)[1] = Q;
    INTEGER(dim)[2] = B;
    dimgets(ans, dim);
    RC_ThreeTableSums(INTEGER(x), N, P, INTEGER(y), Q,
                      INTEGER(block), B,
                      weights, subset, Offset0, Nsubset, REAL(ans));
    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 117a.
Defines: `R_ThreeTableSums` 16, 127a, 164, 165.
Uses: `B` 28c, `block` 28bd, `mPQB` 141a, `N` 24bc, `NLEVELS` 140a, `Nsubset` 27c, `Offset0` 22b, `P` 25a, `Q` 25e, `RC_ThreeTableSums` 128b, `subset` 27be, 28a, `weights` 26c, `weights`, 26de, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *RC_ThreeTableSums Prototype* 128a ⟩ ≡

```
void RC_ThreeTableSums
(
    ⟨ C ThreeTableSums Input 128c ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ C subset range Input 27d ⟩,
    ⟨ C ThreeTableSums Answer 129a ⟩
)
◇
```

Fragment referenced in 128b.
Uses: `RC_ThreeTableSums` 128b.

⟨ *RC_ThreeTableSums* 128b ⟩ ≡

```
⟨ RC_ThreeTableSums Prototype 128a ⟩
{
    if (TYPEOF(weights) == INTSXP) {
        if (TYPEOF(subset) == INTSXP) {
            C_ThreeTableSums_iweights_isubset(x, N, P, y, Q, block, B,
                                            INTEGER(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                            offset, Nsubset, PQL_ans);
        } else {
            C_ThreeTableSums_iweights_dsubset(x, N, P, y, Q, block, B,
                                            INTEGER(weights), XLENGTH(weights) > 0, REAL(subset),
                                            offset, Nsubset, PQL_ans);
        }
    } else {
        if (TYPEOF(subset) == INTSXP) {
            C_ThreeTableSums_dweights_isubset(x, N, P, y, Q, block, B,
                                            REAL(weights), XLENGTH(weights) > 0, INTEGER(subset),
                                            offset, Nsubset, PQL_ans);
        } else {
            C_ThreeTableSums_dweights_dsubset(x, N, P, y, Q, block, B,
                                            REAL(weights), XLENGTH(weights) > 0, REAL(subset),
                                            offset, Nsubset, PQL_ans);
        }
    }
}
◇
```

Fragment referenced in 117a.
Defines: `RC_ThreeTableSums` 44, 127b, 128a.
Uses: `B` 28c, `block` 28bd, `C_ThreeTableSums_dweights_dsubset` 129b, `C_ThreeTableSums_dweights_isubset` 130b, `C_ThreeTableSums_iweights_dsubset` 129c, `C_ThreeTableSums_iweights_isubset` 130a, `N` 24bc, `Nsubset` 27c, `offset` 27d, `P` 25a, `Q` 25e, `subset` 27be, 28a, `weights` 26c, `x` 24d, 25bc, `y` 25d, 26ab.

⟨ *C ThreeTableSums Input* 128c ⟩ ≡

```
⟨ C integer x Input 25c ⟩
⟨ C integer y Input 26b ⟩
⟨ C integer block Input 28d ⟩
◇
```

Fragment referenced in 128a, 129bc, 130ab.

⟨ *C ThreeTableSums Answer* 129a ⟩ ≡

```
double *PQL_ans
```
◇

Fragment referenced in 128a, 129bc, 130ab.

⟨ *C_ThreeTableSums_dweights_dsubset* 129b ⟩ ≡

```
void C_ThreeTableSums_dweights_dsubset
(
    ⟨ C ThreeTableSums Input 128c ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C ThreeTableSums Answer 129a ⟩
) {
    double *s, *w;
    ⟨ ThreeTableSums Body 131a ⟩
}
```
◇

Fragment referenced in 117a.
Defines: `C_ThreeTableSums_dweights_dsubset` 128b.

⟨ *C_ThreeTableSums_iweights_dsubset* 129c ⟩ ≡

```
void C_ThreeTableSums_iweights_dsubset
(
    ⟨ C ThreeTableSums Input 128c ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C real subset Input 28a ⟩,
    ⟨ C ThreeTableSums Answer 129a ⟩
) {
    double *s;
    int *w;
    ⟨ ThreeTableSums Body 131a ⟩
}
```
◇

Fragment referenced in 117a.
Defines: `C_ThreeTableSums_iweights_dsubset` 128b.

⟨ *C_ThreeTableSums_iweights_isubset* 130a ⟩ ≡

```
void C_ThreeTableSums_iweights_isubset
(
    ⟨ C ThreeTableSums Input 128c ⟩
    ⟨ C integer weights Input 26d ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C ThreeTableSums Answer 129a ⟩
) {
    int *s, *w;
    ⟨ ThreeTableSums Body 131a ⟩
}
◇
```

Fragment referenced in 117a.
Defines: `C_ThreeTableSums_iweights_isubset` 128b.

⟨ *C_ThreeTableSums_dweights_isubset* 130b ⟩ ≡

```
void C_ThreeTableSums_dweights_isubset
(
    ⟨ C ThreeTableSums Input 128c ⟩
    ⟨ C real weights Input 26e ⟩
    ⟨ C integer subset Input 27e ⟩,
    ⟨ C ThreeTableSums Answer 129a ⟩
) {
    int *s;
    double *w;
    ⟨ ThreeTableSums Body 131a ⟩
}
◇
```

Fragment referenced in 117a.
Defines: `C_ThreeTableSums_dweights_isubset` 128b.

⟨ *ThreeTableSums Body* 131a ⟩ ≡

```
int *xx, *yy, *bb, PQ = mPQB(P, Q, 1);

for (int p = 0; p < PQ * B; p++) PQL_ans[p] = 0.0;

yy = y;
xx = x;
bb = block;
⟨ init subset loop 93b ⟩
⟨ start subset loop 94a ⟩
{
    xx = xx + diff;
    yy = yy + diff;
    bb = bb + diff;
    if (HAS_WEIGHTS) {
        w = w + diff;
        PQL_ans[(bb[0] - 1) * PQ + yy[0] * P + xx[0]] += (double) w[0];
    } else {
        PQL_ans[(bb[0] - 1) * PQ + yy[0] * P + xx[0]]++;
    }
    ⟨ continue subset loop 94b ⟩
}
xx = xx + diff;
yy = yy + diff;
bb = bb + diff;
if (HAS_WEIGHTS) {
    w = w + diff;
    PQL_ans[(bb[0] - 1) * PQ + yy[0] * P + xx[0]] += w[0];
} else {
    PQL_ans[(bb[0] - 1) * PQ + yy[0] * P + xx[0]]++;
}
◇
```

Fragment referenced in 129bc, 130ab.
Uses: `B` 28c, `block` 28bd, `HAS_WEIGHTS` 26de, `mPQB` 141a, `P` 25a, `Q` 25e, `x` 24d, 25bc, `y` 25d, 26ab.

3.10 Utilities

3.10.1 Blocks

```
> sb <- sample(block)
> ns1 <- do.call("c", tapply(subset, sb[subset], function(i) i))
> ns2 <- .Call(libcoin:::R_order_subset_wrt_block, y, integer(0), subset, sb)
> stopifnot(isequal(ns1, ns2))
```

⟨ *Utils* 131b ⟩ ≡

```
⟨ C_setup_subset 134a ⟩
⟨ C_setup_subset_block 134b ⟩
⟨ C_order_subset_wrt_block 135a ⟩
⟨ RC_order_subset_wrt_block 133b ⟩
⟨ R_order_subset_wrt_block 132b ⟩
◇
```

Fragment referenced in 24a.

⟨ *R_order_subset_wrt_block Prototype* 132a ⟩ ≡

```
SEXP R_order_subset_wrt_block
(
    ⟨ R y Input 25d ⟩
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ R block Input 28b ⟩
)
◇
```

Fragment referenced in 23b, 132b.
Uses: `R_order_subset_wrt_block` 132b.

⟨ *R_order_subset_wrt_block* 132b ⟩ ≡

```
⟨ R_order_subset_wrt_block Prototype 132a ⟩
{
    ⟨ C integer N Input 24c ⟩;
    SEXP blockTable, ans;

    N = XLENGTH(y) / NCOL(y);

    if (XLENGTH(weights) > 0)
        error("cannot deal with weights here");

    if (NLEVELS(block) > 1) {
        PROTECT(blockTable = R_OneTableSums(block, weights, subset));
    } else {
        PROTECT(blockTable = allocVector(REALSXP, 2));
        REAL(blockTable)[0] = 0.0;
        REAL(blockTable)[1] = RC_Sums(N, weights, subset, Offset0, XLENGTH(subset));
    }

    PROTECT(ans = RC_order_subset_wrt_block(N, subset, block, blockTable));

    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 131b.
Defines: `R_order_subset_wrt_block` 132a, 164, 165.
Uses: `block` 28bd, `blockTable` 28e, `N` 24bc, `NCOL` 139c, `NLEVELS` 140a, `Offset0` 22b, `RC_order_subset_wrt_block` 133b, `RC_Sums` 96a, `R_OneTableSums` 118a, `subset` 27be, 28a, `weights` 26c, `weights`, 26de, `y` 25d, 26ab.

⟨ *RC_order_subset_wrt_block Prototype* 133a ⟩ ≡

```
SEXP RC_order_subset_wrt_block
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ R subset Input 27b ⟩,
    ⟨ R block Input 28b ⟩,
    ⟨ R blockTable Input 28e ⟩
)
◇
```

Fragment referenced in 133b.
Uses: `RC_order_subset_wrt_block` 133b.

⟨ *RC_order_subset_wrt_block* 133b ⟩ ≡

```
⟨ RC_order_subset_wrt_block Prototype 133a ⟩
{
    SEXP ans;
    int NOBLOCK = (XLENGTH(block) == 0 || XLENGTH(blockTable) == 2);

    if (XLENGTH(subset) > 0) {
        if (NOBLOCK) {
            return(subset);
        } else {
            PROTECT(ans = allocVector(TYPEOF(subset), XLENGTH(subset)));
            C_order_subset_wrt_block(subset, block, blockTable, ans);
            UNPROTECT(1);
            return(ans);
        }
    } else {
        PROTECT(ans = allocVector(TYPEOF(subset), N));
        if (NOBLOCK) {
            C_setup_subset(N, ans);
        } else {
            C_setup_subset_block(N, block, blockTable, ans);
        }
        UNPROTECT(1);
        return(ans);
    }
}
◇
```

Fragment referenced in 131b.
Defines: `RC_order_subset_wrt_block` 36a, 40, 132b, 133a.
Uses: `block` 28bd, `blockTable` 28e, `C_order_subset_wrt_block` 135a, `C_setup_subset` 134a, `C_setup_subset_block` 134b, `N` 24bc, `subset` 27be, 28a.

⟨ *C_setup_subset* 134a ⟩ ≡

```
void C_setup_subset
(
    ⟨ C integer N Input 24c ⟩,
    SEXP ans
) {
    for (R_xlen_t i = 0; i < N; i++) {
        /* ans is R style index in 1:N */
        if (TYPEOF(ans) == INTSXP) {
            INTEGER(ans)[i] = i + 1;
        } else {
            REAL(ans)[i] = (double) i + 1;
        }
    }
}
◇
```

Fragment referenced in 131b.
Defines: `C_setup_subset` 133b, 136a.
Uses: `N` 24bc.

⟨ *C_setup_subset_block* 134b ⟩ ≡

```
void C_setup_subset_block
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ R block Input 28b ⟩,
    ⟨ R blockTable Input 28e ⟩,
    SEXP ans
) {
    double *cumtable;
    int Nlevels = LENGTH(blockTable);

    cumtable = Calloc(Nlevels, double);
    for (int k = 0; k < Nlevels; k++) cumtable[k] = 0.0;

    /* table[0] are missings, ie block == 0 ! */
    for (int k = 1; k < Nlevels; k++)
        cumtable[k] = cumtable[k - 1] + REAL(blockTable)[k - 1];

    for (R_xlen_t i = 0; i < N; i++) {
        /* ans is R style index in 1:N */
        if (TYPEOF(ans) == INTSXP) {
            INTEGER(ans)[(int) cumtable[INTEGER(block)[i]]++] = i + 1;
        } else {
            REAL(ans)[(R_xlen_t) cumtable[INTEGER(block)[i]]++] = (double) i + 1;
        }
    }

    Free(cumtable);
}
◇
```

Fragment referenced in 131b.
Defines: `C_setup_subset_block` 133b.
Uses: `block` 28bd, `blockTable` 28e, `N` 24bc.

⟨ *C_order_subset_wrt_block* 135a ⟩ ≡

```
void C_order_subset_wrt_block
(
    ⟨ R subset Input 27b ⟩,
    ⟨ R block Input 28b ⟩,
    ⟨ R blockTable Input 28e ⟩,
    SEXP ans
) {
    double *cumtable;
    int Nlevels = LENGTH(blockTable);

    cumtable = Calloc(Nlevels, double);
    for (int k = 0; k < Nlevels; k++) cumtable[k] = 0.0;

    /* table[0] are missings, ie block == 0 ! */
    for (int k = 1; k < Nlevels; k++)
        cumtable[k] = cumtable[k - 1] + REAL(blockTable)[k - 1];

    /* subset is R style index in 1:N */
    if (TYPEOF(subset) == INTSXP) {
        for (R_xlen_t i = 0; i < XLENGTH(subset); i++)
            INTEGER(ans)[(int) cumtable[INTEGER(block)[INTEGER(subset)[i] -
1]]++] = INTEGER(subset)[i];
    } else {
        for (R_xlen_t i = 0; i < XLENGTH(subset); i++)
            REAL(ans)[(R_xlen_t) cumtable[INTEGER(block)[(R_xlen_t) REAL(subset)[i] -
1]]++] = REAL(subset)[i];
    }

    Free(cumtable);
}
◇
```

Fragment referenced in 131b.
Defines: `C_order_subset_wrt_block` 133b.
Uses: `block` 28bd, `blockTable` 28e, `N` 24bc, `subset` 27be, 28a.

⟨ *RC_setup_subset Prototype* 135b ⟩ ≡

```
SEXP RC_setup_subset
(
    ⟨ C integer N Input 24c ⟩,
    ⟨ R weights Input 26c ⟩,
    ⟨ R subset Input 27b ⟩
)
◇
```

Fragment referenced in 136a.
Uses: `RC_setup_subset` 136a.

Because this will only be used when really needed (in Permutations) we can be a little bit more generous with memory here. The return value is always `REALSXP`.

⟨ *RC_setup_subset* 136a ⟩ ≡

```
⟨ RC_setup_subset Prototype 135b ⟩
{
    SEXP ans, mysubset;
    R_xlen_t sumweights;

    if (XLENGTH(subset) == 0) {
        PROTECT(mysubset = allocVector(REALSXP, N));
        C_setup_subset(N, mysubset);
    } else {
        PROTECT(mysubset = coerceVector(subset, REALSXP));
    }

    if (XLENGTH(weights) == 0) {
        UNPROTECT(1);
        return(mysubset);
    }

    sumweights = (R_xlen_t) RC_Sums(N, weights, mysubset, Offset0, XLENGTH(subset));
    PROTECT(ans = allocVector(REALSXP, sumweights));

    R_xlen_t itmp = 0;
    for (R_xlen_t i = 0; i < XLENGTH(mysubset); i++) {
        if (TYPEOF(weights) == REALSXP) {
            for (R_xlen_t j = 0; j < REAL(weights)[(R_xlen_t) REAL(mysubset)[i] - 1]; j++)
                REAL(ans)[itmp++] = REAL(mysubset)[i];
        } else {
            for (R_xlen_t j = 0; j < INTEGER(weights)[(R_xlen_t) REAL(mysubset)[i] - 1]; j++)
                REAL(ans)[itmp++] = REAL(mysubset)[i];
        }
    }
    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 136b.
Defines: `RC_setup_subset` 40, 135b.
Uses: `C_setup_subset` 134a, `N` 24bc, `Offset0` 22b, `RC_Sums` 96a, `subset` 27be, 28a, `sumweights` 27a, `weights` 26c, `weights,` 26de.

3.10.2 Permutation Helpers

⟨ *Permutations* 136b ⟩ ≡

```
⟨ RC_setup_subset 136a ⟩
⟨ C_Permute 137a ⟩
⟨ C_doPermute 137b ⟩
⟨ C_PermuteBlock 138a ⟩
⟨ C_doPermuteBlock 138b ⟩
◇
```

Fragment referenced in 24a.

⟨ *C_Permute* 137a ⟩ ≡

```
void C_Permute
(
    double *subset,
    ⟨ C integer Nsubset Input 27c ⟩,
    double *ans
) {
    R_xlen_t n = Nsubset, j;

    for (R_xlen_t i = 0; i < Nsubset; i++) {
        j = n * unif_rand();
        ans[i] = subset[j];
        subset[j] = subset[--n];
    }
}
◇
```

Fragment referenced in 136b.
Defines: `C_Permute` 137b, 138a.
Uses: `Nsubset` 27c, `subset` 27be, 28a.

⟨ *C_doPermute* 137b ⟩ ≡

```
void C_doPermute
(
    double *subset,
    ⟨ C integer Nsubset Input 27c ⟩,
    double *Nsubset_tmp,
    double *perm
) {
    Memcpy(Nsubset_tmp, subset, Nsubset);
    C_Permute(Nsubset_tmp, Nsubset, perm);
}
◇
```

Fragment referenced in 136b.
Defines: `C_doPermute` 40.
Uses: `C_Permute` 137a, `Nsubset` 27c, `subset` 27be, 28a.

⟨ *C_PermuteBlock* 138a ⟩ ≡

```
void C_PermuteBlock
(
    double *subset,
    double *table,
    int Nlevels,
    double *ans
) {
    double *px, *pans;

    px = subset;
    pans = ans;

    for (R_xlen_t j = 0; j < Nlevels; j++) {
        if (table[j] > 0) {
            C_Permute(px, (R_xlen_t) table[j], pans);
            px += (R_xlen_t) table[j];
            pans += (R_xlen_t) table[j];
        }
    }
}
◇
```

Fragment referenced in 136b.
Defines: `C_PermuteBlock` 138b.
Uses: `C_Permute` 137a, `subset` 27be, 28a.

⟨ *C_doPermuteBlock* 138b ⟩ ≡

```
void C_doPermuteBlock
(
    double *subset,
    ⟨ C integer Nsubset Input 27c ⟩,
    double *table,
    int Nlevels,
    double *Nsubset_tmp,
    double *perm
) {
    Memcpy(Nsubset_tmp, subset, Nsubset);
    C_PermuteBlock(Nsubset_tmp, table, Nlevels, perm);
}
◇
```

Fragment referenced in 136b.
Defines: `C_doPermuteBlock` 40.
Uses: `C_PermuteBlock` 138a, `Nsubset` 27c, `subset` 27be, 28a.

3.10.3 Other Utils

⟨ *MoreUtils* 139a ⟩ ≡

⟨ *NROW* 139b ⟩
⟨ *NCOL* 139c ⟩
⟨ *NLEVELS* 140a ⟩
⟨ *C_kronecker* 143 ⟩
⟨ *R_kronecker* 142 ⟩
⟨ *C_kronecker_sym* 144 ⟩
⟨ *C_KronSums_sym* 145a ⟩
⟨ *C_MPinv_sym* 147 ⟩
⟨ *R_MPinv_sym* 146b ⟩
⟨ *R_unpack_sym* 149 ⟩
⟨ *R_pack_sym* 150c ⟩
◇

Fragment referenced in 24a.

⟨ *NROW* 139b ⟩ ≡

```
int NROW
(
    SEXP x
) {
    SEXP a;
    a = getAttrib(x, R_DimSymbol);
    if (a == R_NilValue) return(XLENGTH(x));
    if (TYPEOF(a) == REALSXP)
        return(REAL(a)[0]);
    return(INTEGER(a)[0]);
}
```
◇

Fragment referenced in 139a.
Defines: `NROW` 6, 8, 9ab, 14, 35a, 40, 46c, 47, 64c, 140a, 142, 150c.
Uses: `x` 24d, 25bc.

⟨ *NCOL* 139c ⟩ ≡

```
int NCOL
(
    SEXP x
) {
    SEXP a;
    a = getAttrib(x, R_DimSymbol);
    if (a == R_NilValue) return(1);
    if (TYPEOF(a) == REALSXP)
        return(REAL(a)[1]);
    return(INTEGER(a)[1]);
}
```
◇

Fragment referenced in 139a.
Defines: `NCOL` 12, 33, 45a, 64c, 85b, 87a, 100a, 109b, 113b, 132b, 142.
Uses: `x` 24d, 25bc.

⟨ *NLEVELS* 140a ⟩ ≡

```
int NLEVELS
(
    SEXP x
) {
    SEXP a;
    int maxlev = 0;

    a = getAttrib(x, R_LevelsSymbol);
    if (a == R_NilValue) {
        if (TYPEOF(x) != INTSXP)
            error("cannot determine number of levels");
        for (R_xlen_t i = 0; i < XLENGTH(x); i++) {
            if (INTEGER(x)[i] > maxlev)
                maxlev = INTEGER(x)[i];
        }
        return(maxlev);
    }
    return(NROW(a));
}
◇
```

Fragment referenced in 139a.
Defines: `NLEVELS` 33, 45a, 118a, 122b, 127b, 132b.
Uses: `NROW` 139b, `x` 24d, 25bc.

Check for integer overflow when computing $P(P+1)/2$ and PQ.

⟨ *PP12* 140b ⟩ ≡

```
int PP12
(
    int P
) {
    double dP = (double) P;
    double ans;

    ans = dP * (dP + 1) / 2;

    if (ans > INT_MAX)
        error("cannot allocate memory: number of levels too large");

    return((int) ans);
}
◇
```

Fragment referenced in 151a.
Defines: `PP12` 36a, 47, 49, 55, 83, 93a, 159, 160a.
Uses: `P` 25a.

⟨ *mPQB* 141a ⟩ ≡

```
int mPQB
(
    int P,
    int Q,
    int B
) {
    double ans = P * Q * B;

    if (ans > INT_MAX)
        error("cannot allocate memory: number of levels too large");

    return((int) ans);
}
◇
```

Fragment referenced in 151a.
Defines: `mPQB` 38b, 40, 48, 51, 56a, 74, 76a, 80b, 82b, 83, 84, 108, 112b, 122b, 127b, 131a, 159.
Uses: `B` 28c, `P` 25a, `Q` 25e.

```
> A <- matrix(runif(12), ncol = 3)
> B <- matrix(runif(10), ncol = 2)
> K1 <- kronecker(A, B)
> K2 <- .Call(libcoin:::R_kronecker, A, B)
> stopifnot(isequal(K1, K2))
```

`"libcoinAPI.h"` 141b≡

```
extern SEXP libcoin_R_kronecker(
    SEXP A, SEXP B
) {
    static SEXP(*fun)(SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP))
            R_GetCCallable("libcoin", "R_kronecker");
    return fun(A, B);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: `B` 28c.

⟨ *R_kronecker Prototype* 141c ⟩ ≡

```
SEXP R_kronecker
(
    SEXP A,
    SEXP B
)
◇
```

Fragment referenced in 23b, 142.
Uses: `B` 28c.

⟨ *R_kronecker* 142 ⟩ ≡

⟨ *R_kronecker Prototype* 141c ⟩
```
{
    int m, n, r, s;
    SEXP ans;

    if (!isReal(A) || !isReal(B))
        error("R_kronecker: A and / or B are not of type REALSXP");

    m = NROW(A);
    n = NCOL(A);
    r = NROW(B);
    s = NCOL(B);

    PROTECT(ans = allocMatrix(REALSXP, m * n, r * s));
    C_kronecker(REAL(A), m, n, REAL(B), r, s, 1, REAL(ans));
    UNPROTECT(1);
    return(ans);
}
```
◇

Fragment referenced in 139a.
Uses: `B` 28c, `C_kronecker` 143, `NCOL` 139c, `NROW` 139b.

⟨ *C_kronecker* 143 ⟩ ≡

```
void C_kronecker
(
    const double *A,
    const int m,
    const int n,
    const double *B,
    const int r,
    const int s,
    const int overwrite,
    double *ans
) {
    int i, j, k, l, mr, js, ir;
    double y;

    if (overwrite) {
        for (i = 0; i < m * r * n * s; i++) ans[i] = 0.0;
    }

    mr = m * r;
    for (i = 0; i < m; i++) {
        ir = i * r;
        for (j = 0; j < n; j++) {
            js = j * s;
            y = A[j*m + i];
            for (k = 0; k < r; k++) {
                for (l = 0; l < s; l++)
                    ans[(js + l) * mr + ir + k] += y * B[l * r + k];
            }
        }
    }
}
```

◇

Fragment referenced in 139a.
Defines: `C_kronecker` 84, 142.
Uses: `B` 28c, `y` 25d, 26ab.

⟨ *C_kronecker_sym* 144 ⟩ ≡

```
void C_kronecker_sym
(
    const double *A,
    const int m,
    const double *B,
    const int r,
    const int overwrite,
    double *ans
) {
    int i, j, k, l, mr, js, ir, s;
    double y;

    mr = m * r;
    s = r;

    if (overwrite) {
        for (i = 0; i < mr * (mr + 1) / 2; i++) ans[i] = 0.0;
    }

    for (i = 0; i < m; i++) {
        ir = i * r;
        for (j = 0; j <= i; j++) {
            js = j * s;
            y = A[S(i, j, m)];
            for (k = 0; k < r; k++) {
                for (l = 0; l < (j < i ? s : k + 1); l++) {
                    ans[S(ir + k, js + l, mr)] += y * B[S(k, l, r)];
                }
            }
        }
    }
}
◇
```

Fragment referenced in 139a.
Defines: `C_kronecker_sym` 83.
Uses: `B` 28c, `S` 22a, `y` 25d, 26ab.

⟨ *C_KronSums_sym* 145a ⟩ ≡

```
/* sum_i (t(x[i,]) %*% x[i,]) */
void C_KronSums_sym_
(
    ⟨ C real x Input 25b ⟩
    double *PP_sym_ans
) {
    int pN, qN, SpqP;

    for (int q = 0; q < P; q++) {
        qN = q * N;
        for (int p = 0; p <= q; p++) {
            PP_sym_ans[S(p, q, P)] = 0.0;
            pN = p * N;
            SpqP = S(p, q, P);
            for (int i = 0; i < N; i++)
                 PP_sym_ans[SpqP] += x[qN + i] * x[pN + i];
        }
    }
}
◇
```

Fragment referenced in 139a.
Defines: `C_KronSums_sym` Never used.
Uses: `N` 24bc, `P` 25a, `S` 22a, `x` 24d, 25bc.

```
> covar <- vcov(ls1)
> covar_sym <- ls1$Covariance
> n <- (sqrt(1 + 8 * length(covar_sym)) - 1) / 2
> tol <- sqrt(.Machine$double.eps)
> MP1 <- MPinverse(covar, tol)
> MP2 <- .Call(libcoin:::R_MPinv_sym, covar_sym, as.integer(n), tol)
> lt <- lower.tri(covar, diag = TRUE)
> stopifnot(isequal(MP1$MPinv[lt], MP2$MPinv) &&
+            isequal(MP1$rank, MP2$rank))
```

"libcoinAPI.h" 145b≡

```
extern SEXP libcoin_R_MPinv_sym(
    SEXP x, SEXP n, SEXP tol
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_MPinv_sym");
    return fun(x, n, tol);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: `R_MPinv_sym` 146b, `x` 24d, 25bc.

⟨ *R_MPinv_sym Prototype* 146a ⟩ ≡

```
SEXP R_MPinv_sym
(
    SEXP x,
    SEXP n,
    SEXP tol
)
◇
```

Fragment referenced in 23b, 146b.
Uses: `R_MPinv_sym` 146b, `x` 24d, 25bc.

⟨ *R_MPinv_sym* 146b ⟩ ≡

```
⟨ R_MPinv_sym Prototype 146a ⟩
{
    SEXP ans, names, MPinv, rank;

    PROTECT(ans = allocVector(VECSXP, 2));
    PROTECT(names = allocVector(STRSXP, 2));
    SET_VECTOR_ELT(ans, 0, MPinv = allocVector(REALSXP, LENGTH(x)));
    SET_STRING_ELT(names, 0, mkChar("MPinv"));
    SET_VECTOR_ELT(ans, 1, rank = allocVector(INTSXP, 1));
    SET_STRING_ELT(names, 1, mkChar("rank"));
    namesgets(ans, names);

    C_MPinv_sym(REAL(x), INTEGER(n)[0], REAL(tol)[0], REAL(MPinv), INTEGER(rank));

    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 139a.
Defines: `R_MPinv_sym` 145b, 146a, 164, 165.
Uses: `x` 24d, 25bc.

⟨ *C_MPinv_sym* 147 ⟩ ≡

```
void C_MPinv_sym
(
    const double *x,
    const int n,
    const double tol,
    double *dMP,
    int *rank
) {
    double *val, *vec, dtol, *rx, *work, valinv;
    int valzero = 0, info = 0, kn;

    if (n == 1) {
        if (x[0] > tol) {
            dMP[0] = 1 / x[0];
            rank[0] = 1;
        } else {
            dMP[0] = 0;
            rank[0] = 0;
        }
    } else {
        rx = Calloc(n * (n + 1) / 2, double);
        Memcpy(rx, x, n * (n + 1) / 2);
        work = Calloc(3 * n, double);
        val = Calloc(n, double);
        vec = Calloc(n * n, double);

        F77_CALL(dspev)("V", "L", &n, rx, val, vec, &n, work,
                        &info);

        dtol = val[n - 1] * tol;

        for (int k = 0; k < n; k++)
            valzero += (val[k] < dtol);
        rank[0] = n - valzero;

        for (int k = 0; k < n * (n + 1) / 2; k++) dMP[k] = 0.0;

        for (int k = valzero; k < n; k++) {
            valinv = 1 / val[k];
            kn = k * n;
            for (int i = 0; i < n; i++) {
                for (int j = 0; j <= i; j++) {
                    /* MP is symmetric */
                    dMP[S(i, j, n)] += valinv * vec[kn + i] * vec[kn + j];
                }
            }
        }
        Free(rx); Free(work); Free(val); Free(vec);
    }
}
◇
```

Fragment referenced in 139a.
Uses: S 22a, x 24d, 25bc.

```
> m <- matrix(c(3, 2, 1,
```

```
+                2, 4, 2,
+                1, 2, 5),
+              ncol = 3)
> s <- m[lower.tri(m, diag = TRUE)]
> u1 <- .Call(libcoin:::R_unpack_sym, s, NULL, 0L)
> u2 <- .Call(libcoin:::R_unpack_sym, s, NULL, 1L)
> stopifnot(isequal(m, u1) && isequal(diag(m), u2))
```

"libcoinAPI.h" 148a≡

```
extern SEXP libcoin_R_unpack_sym(
    SEXP x, SEXP names, SEXP diagonly
) {
    static SEXP(*fun)(SEXP, SEXP, SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP, SEXP, SEXP))
            R_GetCCallable("libcoin", "R_unpack_sym");
    return fun(x, names, diagonly);
}
◇
```

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: `R_unpack_sym` 149, `x` 24d, 25bc.

⟨ *R_unpack_sym Prototype* 148b ⟩ ≡

```
SEXP R_unpack_sym
(
    SEXP x,
    SEXP names,
    SEXP diagonly
)
◇
```

Fragment referenced in 23b, 149.
Uses: `R_unpack_sym` 149, `x` 24d, 25bc.

⟨ *R_unpack_sym* 149 ⟩ ≡

```
⟨ R_unpack_sym Prototype 148b ⟩
{
    R_xlen_t n, k = 0;
    SEXP ans, dimnames;
    double *dx, *dans;

    // m = n * (n + 1)/2 <=> n^2 + n - 2 * m = 0
    n = sqrt(0.25 + 2 * XLENGTH(x)) - 0.5;

    dx = REAL(x);
    if (INTEGER(diagonly)[0]) {
        PROTECT(ans = allocVector(REALSXP, n));
        if (names != R_NilValue) {
            namesgets(ans, names);
        }
        dans = REAL(ans);
        for (R_xlen_t i = 0; i < n; i++) {
            dans[i] = dx[k];
            k += n - i;
        }
    } else {
        PROTECT(ans = allocMatrix(REALSXP, n, n));
        if (names != R_NilValue) {
            PROTECT(dimnames = allocVector(VECSXP, 2));
            SET_VECTOR_ELT(dimnames, 0, names);
            SET_VECTOR_ELT(dimnames, 1, names);
            dimnamesgets(ans, dimnames);
            UNPROTECT(1);
        }
        dans = REAL(ans);
        for (R_xlen_t i = 0; i < n; i++) {
            dans[i * n + i] = dx[k];     // diagonal
            k++;
            for (R_xlen_t j = i + 1; j < n; j++) {
                dans[i * n + j] = dx[k]; // lower triangular
                dans[j * n + i] = dx[k]; // upper triangular
                k++;
            }
        }
    }

    UNPROTECT(1);
    return ans;
}
◇
```

Fragment referenced in 139a.
Defines: `R_unpack_sym` 10, 148ab, 164, 165.
Uses: `x` 24d, 25bc.

```
> m <- matrix(c(4, 3, 2, 1,
+               3, 5, 4, 2,
+               2, 4, 6, 5,
+               1, 2, 5, 7),
+             ncol = 4)
> s <- m[lower.tri(m, diag = TRUE)]
```

```
> p <- .Call(libcoin:::R_pack_sym, m)
> stopifnot(isequal(s, p))
```

"libcoinAPI.h" 150a≡

```
extern SEXP libcoin_R_pack_sym(
    SEXP x
) {
    static SEXP(*fun)(SEXP) = NULL;
    if (fun == NULL)
        fun = (SEXP(*)(SEXP))
            R_GetCCallable("libcoin", "R_pack_sym");
    return fun(x);
}
```
◇

File defined by 32a, 38d, 41b, 43b, 50b, 54a, 64a, 141b, 145b, 148a, 150a.
Uses: R_pack_sym 150c, x 24d, 25bc.

⟨ *R_pack_sym Prototype* 150b ⟩ ≡

```
SEXP R_pack_sym
(
    SEXP x
)
```
◇

Fragment referenced in 23b, 150c.
Uses: R_pack_sym 150c, x 24d, 25bc.

⟨ *R_pack_sym* 150c ⟩ ≡

```
⟨ R_pack_sym Prototype 150b ⟩
{
    R_xlen_t n, k = 0;
    SEXP ans;
    double *dx, *dans;

    n = NROW(x);
    dx = REAL(x);
    PROTECT(ans = allocVector(REALSXP, n * (n + 1) / 2));
    dans = REAL(ans);

    for (R_xlen_t i = 0; i < n; i++) {
        for (R_xlen_t j = i; j < n; j++) {
          dans[k] = dx[i * n + j];
          k++;
        }
    }

    UNPROTECT(1);
    return ans;
}
```
◇

Fragment referenced in 139a.
Defines: R_pack_sym 150ab, 164, 165.
Uses: NROW 139b, x 24d, 25bc.

3.11 Memory

⟨ *Memory* 151a ⟩ ≡

⟨ *C_get_P* 151c ⟩
⟨ *C_get_Q* 152a ⟩
⟨ *PP12* 140b ⟩
⟨ *mPQB* 141a ⟩
⟨ *C_get_varonly* 152b ⟩
⟨ *C_get_Xfactor* 152c ⟩
⟨ *C_get_LinearStatistic* 152d ⟩
⟨ *C_get_Expectation* 153a ⟩
⟨ *C_get_Variance* 153b ⟩
⟨ *C_get_Covariance* 154a ⟩
⟨ *C_get_ExpectationX* 154b ⟩
⟨ *C_get_ExpectationInfluence* 154c ⟩
⟨ *C_get_CovarianceInfluence* 155a ⟩
⟨ *C_get_VarianceInfluence* 155b ⟩
⟨ *C_get_TableBlock* 155c ⟩
⟨ *C_get_Sumweights* 156a ⟩
⟨ *C_get_Table* 156b ⟩
⟨ *C_get_dimTable* 156c ⟩
⟨ *C_get_B* 157a ⟩
⟨ *C_get_nresample* 157b ⟩
⟨ *C_get_PermutedLinearStatistic* 157c ⟩
⟨ *C_get_tol* 157d ⟩
⟨ *RC_init_LECV_1d* 160b ⟩
⟨ *RC_init_LECV_2d* 161 ⟩
◇

Fragment referenced in 24a.

⟨ *R LECV Input* 151b ⟩ ≡

```
SEXP LECV
```
◇

Fragment referenced in 54b, 56b, 151c, 152abcd, 153ab, 154abc, 155abc, 156abc, 157abcd.
Defines: `LECV` 41bc, 42a, 55, 56a, 57, 58, 59, 72b, 74, 151c, 152abcd, 153ab, 154abc, 155abc, 156abc, 157abcd.

⟨ *C_get_P* 151c ⟩ ≡

```
int C_get_P
(
    ⟨ R LECV Input 151b ⟩
) {
    return(INTEGER(VECTOR_ELT(LECV, dim_SLOT))[0]);
}
```
◇

Fragment referenced in 151a.
Defines: `C_get_P` 35a, 42a, 49, 56a, 59, 74, 153b, 154a, 157b.
Uses: `dim_SLOT` 22b, `LECV` 151b.

⟨ *C_get_Q* 152a ⟩ ≡

```
int C_get_Q
(
    ⟨ R LECV Input 151b ⟩
) {
    return(INTEGER(VECTOR_ELT(LECV, dim_SLOT))[1]);
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_Q` 35a, 42a, 49, 56a, 74, 153b, 154a, 157b.
Uses: `dim_SLOT` 22b, `LECV` 151b.

⟨ *C_get_varonly* 152b ⟩ ≡

```
int C_get_varonly
(
    ⟨ R LECV Input 151b ⟩
) {
    return(INTEGER(VECTOR_ELT(LECV, varonly_SLOT))[0]);
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_varonly` 34, 36a, 38b, 42a, 47, 48, 49, 56a, 57, 74, 154a.
Uses: `LECV` 151b, `varonly_SLOT` 22b.

⟨ *C_get_Xfactor* 152c ⟩ ≡

```
int C_get_Xfactor
(
    ⟨ R LECV Input 151b ⟩
) {
    return(INTEGER(VECTOR_ELT(LECV, Xfactor_SLOT))[0]);
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_Xfactor` 49.
Uses: `LECV` 151b, `Xfactor_SLOT` 22b.

⟨ *C_get_LinearStatistic* 152d ⟩ ≡

```
double* C_get_LinearStatistic
(
    ⟨ R LECV Input 151b ⟩
) {
    return(REAL(VECTOR_ELT(LECV, LinearStatistic_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_LinearStatistic` 35b, 48, 55, 57, 74, 160a.
Uses: `LECV` 151b, `LinearStatistic_SLOT` 22b.

⟨ *C_get_Expectation* 153a ⟩ ≡

```
double* C_get_Expectation
(
    ⟨ R LECV Input 151b ⟩
) {
    return(REAL(VECTOR_ELT(LECV, Expectation_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_Expectation` 37a, 42a, 46c, 55, 57, 74, 160a.
Uses: `Expectation_SLOT` 22b, `LECV` 151b.

⟨ *C_get_Variance* 153b ⟩ ≡

```
double* C_get_Variance
(
    ⟨ R LECV Input 151b ⟩
) {
    int PQ = C_get_P(LECV) * C_get_Q(LECV);
    double *var, *covar;

    if (isNull(VECTOR_ELT(LECV, Variance_SLOT))) {
        SET_VECTOR_ELT(LECV, Variance_SLOT,
                       allocVector(REALSXP, PQ));
        if (!isNull(VECTOR_ELT(LECV, Covariance_SLOT))) {
            covar = REAL(VECTOR_ELT(LECV, Covariance_SLOT));
            var = REAL(VECTOR_ELT(LECV, Variance_SLOT));
            for (int p = 0; p < PQ; p++)
                var[p] = covar[S(p, p, PQ)];
        }
    }
    return(REAL(VECTOR_ELT(LECV, Variance_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_Variance` 37c, 38b, 42a, 47, 48, 57, 74, 154a, 160a.
Uses: `Covariance_SLOT` 22b, `C_get_P` 151c, `C_get_Q` 152a, `LECV` 151b, `S` 22a, `Variance_SLOT` 22b.

⟨ *C_get_Covariance* 154a ⟩ ≡

```
double* C_get_Covariance
(
    ⟨ R LECV Input 151b ⟩
) {
    int PQ = C_get_P(LECV) * C_get_Q(LECV);
    if (C_get_varonly(LECV) && PQ > 1)
        error("Cannot extract covariance from variance only object");
    if (C_get_varonly(LECV) && PQ == 1)
        return(C_get_Variance(LECV));
    return(REAL(VECTOR_ELT(LECV, Covariance_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_Covariance` 38ab, 42a, 47, 48, 55, 57, 74, 160a.
Uses: `Covariance_SLOT` 22b, `C_get_P` 151c, `C_get_Q` 152a, `C_get_Variance` 153b, `C_get_varonly` 152b, `LECV` 151b.

⟨ *C_get_ExpectationX* 154b ⟩ ≡

```
double* C_get_ExpectationX
(
    ⟨ R LECV Input 151b ⟩
) {
    return(REAL(VECTOR_ELT(LECV, ExpectationX_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_ExpectationX` 36a, 49, 74.
Uses: `ExpectationX_SLOT` 22b, `LECV` 151b.

⟨ *C_get_ExpectationInfluence* 154c ⟩ ≡

```
double* C_get_ExpectationInfluence
(
    ⟨ R LECV Input 151b ⟩
) {
    return(REAL(VECTOR_ELT(LECV, ExpectationInfluence_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_ExpectationInfluence` 36a, 49, 160a.
Uses: `ExpectationInfluence_SLOT` 22b, `LECV` 151b.

⟨ *C_get_CovarianceInfluence* 155a ⟩ ≡

```
double* C_get_CovarianceInfluence
(
    ⟨ R LECV Input 151b ⟩
) {
    return(REAL(VECTOR_ELT(LECV, CovarianceInfluence_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_CovarianceInfluence` 36a, 47, 74, 160a.
Uses: `CovarianceInfluence_SLOT` 22b, `LECV` 151b.

⟨ *C_get_VarianceInfluence* 155b ⟩ ≡

```
double* C_get_VarianceInfluence
(
    ⟨ R LECV Input 151b ⟩
) {
    return(REAL(VECTOR_ELT(LECV, VarianceInfluence_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_VarianceInfluence` 36a, 47, 74, 160a.
Uses: `LECV` 151b, `VarianceInfluence_SLOT` 22b.

⟨ *C_get_TableBlock* 155c ⟩ ≡

```
double* C_get_TableBlock
(
    ⟨ R LECV Input 151b ⟩
) {
    if (VECTOR_ELT(LECV, TableBlock_SLOT) == R_NilValue)
        error("object does not contain table block slot");
    return(REAL(VECTOR_ELT(LECV, TableBlock_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_TableBlock` 36a.
Uses: `block` 28bd, `LECV` 151b, `TableBlock_SLOT` 22b.

⟨ *C_get_Sumweights* 156a ⟩ ≡

```
double* C_get_Sumweights
(
    ⟨ R LECV Input 151b ⟩
) {
    if (VECTOR_ELT(LECV, Sumweights_SLOT) == R_NilValue)
        error("object does not contain sumweights slot");
    return(REAL(VECTOR_ELT(LECV, Sumweights_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_Sumweights` 36a, 49.
Uses: `LECV` 151b, `sumweights` 27a, `Sumweights_SLOT` 22b.

⟨ *C_get_Table* 156b ⟩ ≡

```
double* C_get_Table
(
    ⟨ R LECV Input 151b ⟩
) {
    if (LENGTH(LECV) <= Table_SLOT)
        error("Cannot extract table from object");
    return(REAL(VECTOR_ELT(LECV, Table_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_Table` 44, 49.
Uses: `LECV` 151b, `Table_SLOT` 22b.

⟨ *C_get_dimTable* 156c ⟩ ≡

```
int* C_get_dimTable
(
    ⟨ R LECV Input 151b ⟩
) {
    if (LENGTH(LECV) <= Table_SLOT)
        error("Cannot extract table from object");
    return(INTEGER(getAttrib(VECTOR_ELT(LECV, Table_SLOT),
                             R_DimSymbol)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_dimTable` 49, 157a.
Uses: `LECV` 151b, `Table_SLOT` 22b.

⟨ *C_get_B* 157a ⟩ ≡

```
int C_get_B
(
    ⟨ R LECV Input 151b ⟩
) {
    if (VECTOR_ELT(LECV, TableBlock_SLOT) != R_NilValue)
        return(LENGTH(VECTOR_ELT(LECV, Sumweights_SLOT)));
    return(C_get_dimTable(LECV)[2]);
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_B` 35a, 49, 74.
Uses: `C_get_dimTable` 156c, `LECV` 151b, `Sumweights_SLOT` 22b, `TableBlock_SLOT` 22b.

⟨ *C_get_nresample* 157b ⟩ ≡

```
R_xlen_t C_get_nresample
(
    ⟨ R LECV Input 151b ⟩
) {
    int PQ = C_get_P(LECV) * C_get_Q(LECV);
    return(XLENGTH(VECTOR_ELT(LECV, PermutedLinearStatistic_SLOT)) / PQ);
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_nresample` 42a, 55, 56a, 57, 59, 74.
Uses: `C_get_P` 151c, `C_get_Q` 152a, `LECV` 151b, `PermutedLinearStatistic_SLOT` 22b.

⟨ *C_get_PermutedLinearStatistic* 157c ⟩ ≡

```
double* C_get_PermutedLinearStatistic
(
    ⟨ R LECV Input 151b ⟩
) {
    return(REAL(VECTOR_ELT(LECV, PermutedLinearStatistic_SLOT)));
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_PermutedLinearStatistic` 42a, 55, 57, 74.
Uses: `LECV` 151b, `PermutedLinearStatistic_SLOT` 22b.

⟨ *C_get_tol* 157d ⟩ ≡

```
double C_get_tol
(
    ⟨ R LECV Input 151b ⟩
) {
    return(REAL(VECTOR_ELT(LECV, tol_SLOT))[0]);
}
◇
```

Fragment referenced in 151a.
Defines: `C_get_tol` 42a, 55, 57, 74.
Uses: `LECV` 151b, `tol_SLOT` 22b.

⟨ *Memory Input Checks* 158a ⟩ ≡

```
if (P <= 0)
    error("P is not positive");

if (Q <= 0)
    error("Q is not positive");

if (B <= 0)
    error("B is not positive");

if (varonly < 0 || varonly > 1)
    error("varonly is not 0 or 1");

if (Xfactor < 0 || Xfactor > 1)
    error("Xfactor is not 0 or 1");

if (tol <= DBL_MIN)
    error("tol is not positive");
◇
```

Fragment referenced in 159.
Uses: B 28c, P 25a, Q 25e.

⟨ *Memory Names* 158b ⟩ ≡

```
PROTECT(names = allocVector(STRSXP, Table_SLOT + 1));
SET_STRING_ELT(names, LinearStatistic_SLOT, mkChar("LinearStatistic"));
SET_STRING_ELT(names, Expectation_SLOT, mkChar("Expectation"));
SET_STRING_ELT(names, varonly_SLOT, mkChar("varonly"));
SET_STRING_ELT(names, Variance_SLOT, mkChar("Variance"));
SET_STRING_ELT(names, Covariance_SLOT, mkChar("Covariance"));
SET_STRING_ELT(names, ExpectationX_SLOT, mkChar("ExpectationX"));
SET_STRING_ELT(names, dim_SLOT, mkChar("dimension"));
SET_STRING_ELT(names, ExpectationInfluence_SLOT,
               mkChar("ExpectationInfluence"));
SET_STRING_ELT(names, Xfactor_SLOT, mkChar("Xfactor"));
SET_STRING_ELT(names, CovarianceInfluence_SLOT,
               mkChar("CovarianceInfluence"));
SET_STRING_ELT(names, VarianceInfluence_SLOT,
               mkChar("VarianceInfluence"));
SET_STRING_ELT(names, TableBlock_SLOT, mkChar("TableBlock"));
SET_STRING_ELT(names, Sumweights_SLOT, mkChar("Sumweights"));
SET_STRING_ELT(names, PermutedLinearStatistic_SLOT,
               mkChar("PermutedLinearStatistic"));
SET_STRING_ELT(names, StandardisedPermutedLinearStatistic_SLOT,
               mkChar("StandardisedPermutedLinearStatistic"));
SET_STRING_ELT(names, tol_SLOT, mkChar("tol"));
SET_STRING_ELT(names, Table_SLOT, mkChar("Table"));
◇
```

Fragment referenced in 159.
Uses: CovarianceInfluence_SLOT 22b, Covariance_SLOT 22b, dim_SLOT 22b, ExpectationInfluence_SLOT 22b, ExpectationX_SLOT 22b, Expectation_SLOT 22b, LinearStatistic_SLOT 22b, PermutedLinearStatistic_SLOT 22b, StandardisedPermutedLinearStatistic_SLOT 22b, Sumweights_SLOT 22b, TableBlock_SLOT 22b, Table_SLOT 22b, tol_SLOT 22b, VarianceInfluence_SLOT 22b, Variance_SLOT 22b, varonly_SLOT 22b, Xfactor_SLOT 22b.

⟨ *R_init_LECV* 159 ⟩ ≡

```
SEXP vo, d, names, tolerance, tmp;
int PQ;

⟨ Memory Input Checks 158a ⟩
PQ = mPQB(P, Q, 1);
⟨ Memory Names 158b ⟩

/* Table_SLOT is always last and only used in 2d case, ie omitted here */
PROTECT(ans = allocVector(VECSXP, Table_SLOT + 1));
SET_VECTOR_ELT(ans, LinearStatistic_SLOT, allocVector(REALSXP, PQ));
SET_VECTOR_ELT(ans, Expectation_SLOT, allocVector(REALSXP, PQ));
SET_VECTOR_ELT(ans, varonly_SLOT, vo = allocVector(INTSXP, 1));
INTEGER(vo)[0] = varonly;
if (varonly) {
    SET_VECTOR_ELT(ans, Variance_SLOT, tmp = allocVector(REALSXP, PQ));
} else {
    /* always return variance */
    SET_VECTOR_ELT(ans, Variance_SLOT, tmp = allocVector(REALSXP, PQ));
    SET_VECTOR_ELT(ans, Covariance_SLOT,
                   tmp = allocVector(REALSXP, PP12(PQ)));
}
SET_VECTOR_ELT(ans, ExpectationX_SLOT, allocVector(REALSXP, P));
SET_VECTOR_ELT(ans, dim_SLOT, d = allocVector(INTSXP, 2));
INTEGER(d)[0] = P;
INTEGER(d)[1] = Q;
SET_VECTOR_ELT(ans, ExpectationInfluence_SLOT,
               tmp = allocVector(REALSXP, B * Q));
for (int q = 0; q < B * Q; q++) REAL(tmp)[q] = 0.0;

/* should always _both_ be there */
SET_VECTOR_ELT(ans, VarianceInfluence_SLOT,
               tmp = allocVector(REALSXP, B * Q));
for (int q = 0; q < B * Q; q++) REAL(tmp)[q] = 0.0;

SET_VECTOR_ELT(ans, CovarianceInfluence_SLOT,
               tmp = allocVector(REALSXP, B * Q * (Q + 1) / 2));
for (int q = 0; q < B * Q * (Q + 1) / 2; q++) REAL(tmp)[q] = 0.0;

SET_VECTOR_ELT(ans, Xfactor_SLOT, allocVector(INTSXP, 1));
INTEGER(VECTOR_ELT(ans, Xfactor_SLOT))[0] = Xfactor;
SET_VECTOR_ELT(ans, TableBlock_SLOT, tmp = allocVector(REALSXP, B + 1));
for (int q = 0; q < B + 1; q++) REAL(tmp)[q] = 0.0;
SET_VECTOR_ELT(ans, Sumweights_SLOT, allocVector(REALSXP, B));
SET_VECTOR_ELT(ans, PermutedLinearStatistic_SLOT,
               allocMatrix(REALSXP, 0, 0));
SET_VECTOR_ELT(ans, StandardisedPermutedLinearStatistic_SLOT,
               allocMatrix(REALSXP, 0, 0));
SET_VECTOR_ELT(ans, tol_SLOT, tolerance = allocVector(REALSXP, 1));
REAL(tolerance)[0] = tol;
namesgets(ans, names);

⟨ Initialise Zero 160a ⟩
◇
```

Fragment referenced in 160b, 161.
Uses: B 28c, CovarianceInfluence_SLOT 22b, Covariance_SLOT 22b, dim_SLOT 22b, ExpectationInfluence_SLOT 22b, ExpectationX_SLOT 22b, Expectation_SLOT 22b, LinearStatistic_SLOT 22b, mPQB 141a, P 25a, PermutedLinearStatistic_SLOT 22b, PP12 140b, Q 25e, StandardisedPermutedLinearStatistic_SLOT 22b, Sumweights_SLOT 22b, TableBlock_SLOT 22b, Table_SLOT 22b, tol_SLOT 22b, VarianceInfluence_SLOT 22b, Variance_SLOT 22b, varonly_SLOT 22b, Xfactor_SLOT 22b.

⟨ *Initialise Zero* 160a ⟩ ≡

```
/* set inital zeros */
for (int p = 0; p < PQ; p++) {
    C_get_LinearStatistic(ans)[p] = 0.0;
    C_get_Expectation(ans)[p] = 0.0;
    if (varonly)
        C_get_Variance(ans)[p] = 0.0;
}
if (!varonly) {
    for (int p = 0; p < PP12(PQ); p++)
        C_get_Covariance(ans)[p] = 0.0;
}
for (int q = 0; q < Q; q++) {
    C_get_ExpectationInfluence(ans)[q] = 0.0;
    C_get_VarianceInfluence(ans)[q] = 0.0;
}
for (int q = 0; q < Q * (Q + 1) / 2; q++)
    C_get_CovarianceInfluence(ans)[q] = 0.0;
◇
```

Fragment referenced in 159.
Uses: `C_get_Covariance` 154a, `C_get_CovarianceInfluence` 155a, `C_get_Expectation` 153a, `C_get_ExpectationInfluence` 154c, `C_get_LinearStatistic` 152d, `C_get_Variance` 153b, `C_get_VarianceInfluence` 155b, `PP12` 140b, `Q` 25e.

⟨ *RC_init_LECV_1d* 160b ⟩ ≡

```
SEXP RC_init_LECV_1d
(
    ⟨ C integer P Input 25a ⟩,
    ⟨ C integer Q Input 25e ⟩,
    int varonly,
    ⟨ C integer B Input 28c ⟩,
    int Xfactor,
    double tol
) {
    SEXP ans;

    ⟨ R_init_LECV 159 ⟩

    SET_VECTOR_ELT(ans, TableBlock_SLOT,
                   allocVector(REALSXP, B + 1));

    SET_VECTOR_ELT(ans, Sumweights_SLOT,
                   allocVector(REALSXP, B));

    UNPROTECT(2);
    return(ans);
}
◇
```

Fragment referenced in 151a.
Defines: `RC_init_LECV_1d` 32c.
Uses: `B` 28c, `Sumweights_SLOT` 22b, `TableBlock_SLOT` 22b.

⟨ *RC_init_LECV_2d* 161 ⟩ ≡

```
SEXP RC_init_LECV_2d
(
    ⟨ C integer P Input 25a ⟩,
    ⟨ C integer Q Input 25e ⟩,
    int varonly,
    int Lx,
    int Ly,
    ⟨ C integer B Input 28c ⟩,
    int Xfactor,
    double tol
) {
    SEXP ans, tabdim, tab;

    if (Lx <= 0)
        error("Lx is not positive");

    if (Ly <= 0)
        error("Ly is not positive");

    ⟨ R_init_LECV 159 ⟩

    PROTECT(tabdim = allocVector(INTSXP, 3));
    INTEGER(tabdim)[0] = Lx + 1;
    INTEGER(tabdim)[1] = Ly + 1;
    INTEGER(tabdim)[2] = B;
    SET_VECTOR_ELT(ans, Table_SLOT,
                   tab = allocVector(REALSXP,
                       INTEGER(tabdim)[0] *
                       INTEGER(tabdim)[1] *
                       INTEGER(tabdim)[2]));
    dimgets(tab, tabdim);

    UNPROTECT(3);
    return(ans);
}
◇
```

Fragment referenced in 151a.
Defines: `RC_init_LECV_2d` 44.
Uses: `B` 28c, `Table_SLOT` 22b.

Chapter 4

Package Infrastructure

"AAA.R" 162a≡

```
⟨ R Header 166a ⟩
.onUnload <- function(libpath)
    library.dynam.unload("libcoin", libpath)
◇
```

"DESCRIPTION" 162b≡

```
Package: libcoin
Title: Linear Test Statistics for Permutation Inference
Date: 20YY-MM-DD
Version: 1.0-7
Authors@R: person("Torsten", "Hothorn", role = c("aut", "cre"),
                  email = "Torsten.Hothorn@R-project.org")
Description: Basic infrastructure for linear test statistics and permutation
  inference in the framework of Strasser and Weber (1999) <https://epub.wu.ac.at/102/>.
  This package must not be used by end-users. CRAN package 'coin' implements all
  user interfaces and is ready to be used by anyone.
Depends: R (>= 3.4.0)
Suggests: coin
Imports: stats, mvtnorm
LinkingTo: mvtnorm
NeedsCompilation: yes
License: GPL-2
◇
```

"NAMESPACE" 162c≡

```
useDynLib(libcoin, .registration = TRUE)

importFrom("stats", complete.cases, vcov)
importFrom("mvtnorm", GenzBretz)

export(LinStatExpCov, doTest, ctabs, "lmult")
S3method("vcov", "LinStatExpCov")
◇
```

Add flag -g to PKG_CFLAGS for operf profiling (this is not portable).

"Makevars" 163a≡

```
PKG_CFLAGS=$(C_VISIBILITY)
PKG_LIBS = $(LAPACK_LIBS) $(BLAS_LIBS) $(FLIBS)
◇
```

"libcoin-win.def" 163b≡

```
LIBRARY libcoin.dll
EXPORTS
  R_init_libcoin
◇
```

Other packages can link against **libcoin**. A small example package is contained in libcoin/inst/C_API_example.

"libcoin-init.c" 164≡

```
⟨ C Header 166b ⟩
#include "libcoin.h"
#include <R_ext/Rdynload.h>
#include <R_ext/Visibility.h>

#define CALLDEF(name, n) {#name, (DL_FUNC) &name, n}
#define REGCALL(name) R_RegisterCCallable("libcoin", #name, (DL_FUNC) &name)

static const R_CallMethodDef callMethods[] = {
    CALLDEF(R_ExpectationCovarianceStatistic, 7),
    CALLDEF(R_PermutedLinearStatistic, 6),
    CALLDEF(R_StandardisePermutedLinearStatistic, 1),
    CALLDEF(R_ExpectationCovarianceStatistic_2d, 9),
    CALLDEF(R_PermutedLinearStatistic_2d, 7),
    CALLDEF(R_QuadraticTest, 5),
    CALLDEF(R_MaximumTest, 9),
    CALLDEF(R_MaximallySelectedTest, 6),
    CALLDEF(R_ExpectationInfluence, 3),
    CALLDEF(R_CovarianceInfluence, 4),
    CALLDEF(R_ExpectationX, 4),
    CALLDEF(R_CovarianceX, 5),
    CALLDEF(R_Sums, 3),
    CALLDEF(R_KronSums, 6),
    CALLDEF(R_KronSums_Permutation, 5),
    CALLDEF(R_colSums, 3),
    CALLDEF(R_OneTableSums, 3),
    CALLDEF(R_TwoTableSums, 4),
    CALLDEF(R_ThreeTableSums, 5),
    CALLDEF(R_order_subset_wrt_block, 4),
    CALLDEF(R_quadform, 3),
    CALLDEF(R_kronecker, 2),
    CALLDEF(R_MPinv_sym, 3),
    CALLDEF(R_unpack_sym, 3),
    CALLDEF(R_pack_sym, 1),
    {NULL, NULL, 0}
};
◇
```

File defined by 164, 165.

Uses: R_colSums 113b, R_CovarianceInfluence 87a, R_CovarianceX 92a, R_ExpectationCovarianceStatistic 32c, R_ExpectationCovarianceStatistic_2d 44, R_ExpectationInfluence 85b, R_ExpectationX 89a, R_KronSums 100a, R_KronSums_Permutation 109b, R_MPinv_sym 146b, R_OneTableSums 118a, R_order_subset_wrt_block 132b, R_pack_sym 150c, R_PermutedLinearStatistic 40, R_PermutedLinearStatistic_2d 51, R_quadform 64c, R_Sums 95b, R_ThreeTableSums 127b, R_TwoTableSums 122b, R_unpack_sym 149.

"libcoin-init.c" 165≡

```
void attribute_visible R_init_libcoin
(
    DllInfo *dll
) {
    R_registerRoutines(dll, NULL, callMethods, NULL, NULL);
    R_useDynamicSymbols(dll, FALSE);
    R_forceSymbols(dll, TRUE);
    REGCALL(R_ExpectationCovarianceStatistic);
    REGCALL(R_PermutedLinearStatistic);
    REGCALL(R_StandardisePermutedLinearStatistic);
    REGCALL(R_ExpectationCovarianceStatistic_2d);
    REGCALL(R_PermutedLinearStatistic_2d);
    REGCALL(R_QuadraticTest);
    REGCALL(R_MaximumTest);
    REGCALL(R_MaximallySelectedTest);
    REGCALL(R_ExpectationInfluence);
    REGCALL(R_CovarianceInfluence);
    REGCALL(R_ExpectationX);
    REGCALL(R_CovarianceX);
    REGCALL(R_Sums);
    REGCALL(R_KronSums);
    REGCALL(R_KronSums_Permutation);
    REGCALL(R_colSums);
    REGCALL(R_OneTableSums);
    REGCALL(R_TwoTableSums);
    REGCALL(R_ThreeTableSums);
    REGCALL(R_order_subset_wrt_block);
    REGCALL(R_quadform);
    REGCALL(R_kronecker);
    REGCALL(R_MPinv_sym);
    REGCALL(R_unpack_sym);
    REGCALL(R_pack_sym);
}
◇
```

File defined by 164, 165.
Uses: R_colSums 113b, R_CovarianceInfluence 87a, R_CovarianceX 92a, R_ExpectationCovarianceStatistic 32c, R_ExpectationCovarianceStatistic_2d 44, R_ExpectationInfluence 85b, R_ExpectationX 89a, R_KronSums 100a, R_KronSums_Permutation 109b, R_MPinv_sym 146b, R_OneTableSums 118a, R_order_subset_wrt_block 132b, R_pack_sym 150c, R_PermutedLinearStatistic 40, R_PermutedLinearStatistic_2d 51, R_quadform 64c, R_Sums 95b, R_ThreeTableSums 127b, R_TwoTableSums 122b, R_unpack_sym 149.

⟨ *R Header* 166a ⟩ ≡

```
###     Copyright (C) 2017-2020 Torsten Hothorn
###
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###
###
###     DO NOT EDIT THIS FILE
###
###     Edit 'libcoin.w' and run 'nuweb -r libcoin.w'
```
◇

Fragment referenced in 3a, 16, 162a.

⟨ *C Header* 166b ⟩ ≡

```
/*
    Copyright (C) 2017-2020 Torsten Hothorn

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    DO NOT EDIT THIS FILE

    Edit 'libcoin.w' and run 'nuweb -r libcoin.w'
*/
```
◇

Fragment referenced in 21a, 23ac, 32a, 164.

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Bibliography

Helmut Strasser and Christian Weber. On the asymptotic theory of permutation statistics. *Mathematical Methods of Statistics*, 8:220–250, 1999. preprint available from http://epub.wu-wien.ac.at/dyn/openURL?id=oai:epub.wu-wien.ac.at:epub-wu-01_94c. 1