An Implementation of Negative Binomial 1 Regression in R

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Abstract

An implementation of negative binomial 1 regression models is provided in the **negbin1** package (https://R-Forge.R-project.org/projects/uibk-rprog-2017/). The negative binomial distribution is an often used alternative to the Poisson model, especially when doubts arise regarding the independence of the underlying process and the inclusion of all relevant regressors. The aim of this paper is to give a short overview of the package negbin1, along with an empirical example.

Keywords: negative binomial, NB1, count data, R.

1. Idea

According to Winkelmann (2013), "the negative binomial distribution is the most commonly used alternative to the Poisson model when it is doubtful whether the strict requirements of independence of the underlying process, and inclusion of all relevant regressors, are satisfied." The negative binomial 1 model (NB1) is obtained by letting α vary across individuals such that $\alpha = \sigma^{-2} \exp(x^{\top}\beta)$ and the conditional expectiation function is

$$\mathsf{E}(y|x) = \exp(x^{\top}\beta),\tag{1}$$

which produces a conditional variance that is a linear function of the mean (Winkelmann 2013):

$$\operatorname{Var}(y|x) = (1 + \sigma^2) \cdot \exp(x^{\top}\beta) \tag{2}$$

For more details, see also Cameron and Trivedi (2013).

2. Implementation

The workhorse function of **negbin1** is the function **negbin1_fit**, which is normally not called directly in may other regression packages in R (?), but when the model response and model matrix have already been calculated. Starting values for optimization in **negbin1_fit** are by default taken from Poisson regression. The main model fitting function is **negbin1()** returns an (S3) object of class **negbin1**:

Method	Description
<pre>print()</pre>	Simple printed display with coefficients
<pre>summary()</pre>	Standard regression summary; returns summary.negbin1 object
	(with print() method)
coef()	Extract coefficients
vcov()	Associated covariance matrix
<pre>predict()</pre>	(Different types of) predictions for new data
fitted()	Fitted values for observed data
residuals()	Extract (different types of) residuals
terms()	Extract terms
<pre>model.matrix()</pre>	Extract model matrix (or matrices)
nobs()	Extract number of observations
logLik()	Extract fitted log-likelihood
bread()	Extract bread for sandwich covariance
estfun()	Extract estimating functions (= gradient contributions) for sand-
	wich covariances
getSummary()	Extract summary statistics for mtable()

Table 1: S3 methods provided in **negbin1**.

A number of standard S3 methods are provided, see Table 1. Especially the predict method deserves greater attention:

Due to these methods a number of useful utilities work automatically, e.g., AIC(), BIC(), coeftest() (lmtest), lrtest() (lmtest), waldtest() (lmtest), linearHypothesis() (car), mtable() (memisc), etc.

Predictions

It is also possible to obtain predictions from an object of class negbin. For type = "response" (as well as for type = "location"), the conditional mean, the inverse link applied to the linear predictor, is calculated. type = "probability" computes the expected probabilities for each count at = 0, 1, 2, 3, ..., whereas type = "quantile" gives the quantile function for probabilities at.

```
1 2 3 4
1 3 2 10
R> predict(mod, newdata, type = "probability", at = 0)
1 2 3 4
0.8292225 0.6911623 0.8089649 0.4688839
```

Computation time

Below, the computation time of function negbin1() is compared to NB1 implementations from other packages, as vglm from VGAM (??) and gamlss from package gamlss (?). The simulated dataset comprises 10,000 observations.

	user.self	sys.self	elapsed
<pre>negbin1(grad = FALSE)</pre>	3.596	0.016	3.611
<pre>negbin1(grad = TRUE)</pre>	1.336	0.000	1.336
vglm	5.456	0.000	5.459
gamlss	0.720	0.004	0.724

The computation time of negbin1() does quite well compared to vglm(), but only in case that grad = TRUE. gamlss() is even faster than negbin1() with grad = TRUE.

3. Example

Data on the number of publications produced by Ph.D. biochemists taken from Long (1990) are employed to test the **negbin1** function (see also Scott Long 1997). The data are loaded from the **negbin1** package.

```
R> data("publications", package = "negbin1")
```

In a next step, a benchmark Poisson model is estimated, where the number of publications produced by Ph.D. biochemist appears as a dependent variable. Explanatory variables are gender, marriage status (married), number of children under age six (kids), prestige of Ph.D. program (prestige) and articles by mentor in the last three years (mentor).

```
R> poi <- glm(articles ~ gender + married + kids + prestige + mentor,
+ family = "poisson", data = publications)
R> summary(poi)
Call:
glm(formula = articles ~ gender + married + kids + prestige +
mentor, family = "poisson", data = publications)
Deviance Residuals:
```

```
1Q
                  Median
                               ЗQ
                                       Max
   Min
-3.5672 -1.5398 -0.3660
                           0.5722
                                    5.4467
Coefficients:
               Estimate Std. Error z value Pr(|z|)
(Intercept)
                          0.102981
                                     2.958
                                            0.0031 **
               0.304617
                          0.054613 -4.112 3.92e-05 ***
genderWomen
              -0.224594
marriedMarried 0.155243
                          0.061374 2.529
                                            0.0114 *
kids
              -0.184883
                          0.040127 -4.607 4.08e-06 ***
               0.012823
                          0.026397 0.486
                                            0.6271
prestige
               0.025543
                          0.002006 12.733 < 2e-16 ***
mentor
___
Signif. codes: 0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1
```

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 1817.4 on 914 degrees of freedom Residual deviance: 1634.4 on 909 degrees of freedom AIC: 3314.1

Number of Fisher Scoring iterations: 5

However, the variance of the number of publications is more than twice the mean.

R> var(publications\$articles) / mean(publications\$articles)

[1] 2.191358

As the data exhibit overdispersion (of course without taking any covariates into account), a negative binomial 1 regression offers some remedy in such a situation.

```
R> nb1 <- negbin1(articles ~ gender + married + kids + prestige + mentor,
       data = publications, grad = TRUE)
+
R> summary(nb1)
Call:
negbin1(formula = articles ~ gender + married + kids + prestige + mentor,
    data = publications, grad = TRUE)
Standardized residuals:
     Min
                 1Q
                       Median
                                     ЗQ
                                              Max
            -1.0000
-235.9945
                       1.5878
                                 4.7260 1702.3317
Coefficients:
               Estimate Std. Error z value Pr(>|z|)
(Intercept)
                           0.13222 1.800 0.071882 .
                0.23797
```

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genderWomen -0.18268 0.06985 -2.615 0.008917 ** marriedMarried 0.15667 0.07874 1.990 0.046621 * -0.17297 0.05108 -3.386 0.000709 *** kids 0.928 0.353513 prestige 0.03154 0.03400 0.00260 9.296 < 2e-16 *** mentor 0.02416 log(alpha) -0.23473 0.12278 -1.912 0.055904 . ___ Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 Log-likelihood: -1565 on 7 Df Number of iterations in BFGS optimization: 16

While both sets of parameter estimates would lead to the same conclusions, the standard errors reported reveal that ordinary Poisson regression underestimates the standard errors.

R> cbind("Poisson" = sqrt(diag(vcov(poi))), "NB1" = sqrt(diag(vcov(nb1)))[-7]) Poisson NB1 (Intercept) 0.102981443 0.132217871 genderWomen 0.054613488 0.069853861 marriedMarried 0.061374395 0.078740071 0.040126898 0.051082507 kids prestige 0.026397045 0.033999414 mentor 0.002006073 0.002599541 R> library("memisc") R> mtable("Poisson" = poi, "NB1(negbin1)" = nb1, summary.stats = c("Log-likelihood", "AIC", "BIC", "N")) +

	Poisson	NB1
(Intercept)	0.305^{**}	0.238
	(0.103)	(0.132)
gender: Women/Men	-0.225^{***}	-0.183^{**}
	(0.055)	(0.070)
married: Married/Single	0.155^{*}	0.157^{*}
	(0.061)	(0.079)
kids	-0.185^{***}	-0.173^{***}
	(0.040)	(0.051)
prestige	0.013	0.032
	(0.026)	(0.034)
mentor	0.026^{***}	0.024^{***}
	(0.002)	(0.003)
$\log(alpha)$		-0.235
		(0.123)
Log-likelihood	-1651.1	-1564.7
AIC	3314.1	3143.4
BIC	3343.0	3177.1
Ν	915	915

Table 2: Comparing Poisson results with NB1 from negbin1.

References

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